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REPORTS, PAPERS, DISCUSSIONS, AND MEMOIRS

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UTILIZABLE CAPACITY OF STEEL MEMBERS OF STRUCTURES

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Synopsis

This paper gives the results of a study to determine the fundamental facts on which the utilizable capacity of steel members of structures depends, and to devise methods whereby the utilizable capacity of any member can be determined, as nearly as practicable.

The utilizable capacity of any steel member is measured, as the case may be, by the greatest load which statically or repeatedly, or by the greatest opposite loads which, alternately, can be applied during the life time of the structure (as determined by other considerations) without causing failure, ruin, or seriously objectionable deformation (plastic, elastic, or combined plastic and elastic).

The subject is sub-divided and developed as follows:

Note.—This paper is issued before the date set for presentation and discussion. Correspondence is invited and may be sent by mail to the Secretary. Discussion on the paper will be closed in April, 1926, and, when finally closed, the paper, with discussion in full, will be published in *Transactions*.

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In the systematic development of the subject under the headings noted, pertinent conclusions are successively reached which are briefly stated, as follows:

(*a*) Owing to the presence in plates, shapes, and bars, as they come from the mill yard, of a balanced system of internal forces (herein termed "auto-stresses"), the elastic limit, as determined in a long length with very delicate micrometers and measured by the intensity of the external forces which produce it, is in nearly all cases very low.

(*b*) The elastic limit, strictly defined, would be of no value as a criterion if determined, and should not be specified.

(*c*) In commercial practice a specified "elastic limit" is understood by both the seller and the buyer to be the point in a commercial test, as ordinarily conducted, properly known as the "yield point."

(*d*) The elasticity of a piece of steel, over a wide range of stress, can be perfected, or seemingly perfected, and then remain perfect or seemingly perfect, as nearly as may be gauged by very delicate micrometers, during long continued static or fluctuating stress. The limitations to this range are the criteria for gauging utilizable capacity in cases of alternate tension and compression, but are not critical, at least for untreated steel,* in cases where the stress is always direct tension or direct compression.

(*e*) The point which is the best criterion of utilizable capacity in a short column or tension member not subject to alternating stress, is the point named by the Society's Special Committee on Steel Columns and Struts the "Useful Limit Point"† (U. L. P.), beyond which plastic deformation is a dangerous factor and soon becomes either ruinous or seriously objectionable *per se*. As to just what degree of plastic deformation is seriously objectionable *per se* there is room for some difference of opinion, but in most cases the yield, when well started, develops so rapidly that the zone of room for reasonable difference of opinion is limited. The Committee chose and the writer has adopted as the U. L. P. a point where the rate of combined plastic and elastic deformation reaches twice the rate of elastic deformation.

(*f*) The U. L. P. in a short column or a tension member can be fairly well approximated from the U. L. P.'s and tensile strength of representative specimens by a method described near the end of Part 5.

* Steel which has been treated by cold-rolling, cold-drawing, or quenching, is outside the scope of this paper.

† *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1613.

(*g*) As determined in ordinary commercial tests, the U. L. P.'s cannot be inferred from the yield point, but can be directly determined with fair precision in scientifically conducted tests.

(*h*) When used with judgment, the U. L. P.'s of short, centrally loaded, columns and tension members are the best criteria of utilizable capacity of such columns and members, but tests of centrally loaded medium and long columns are usually made under conditions less severe than in actual service, and indicate the U. L. P.'s which are not the best criteria of utilizable capacity of members in service.

(*i*) The best criterion of the utilizable capacity of a medium column is sound analysis, based (1) on the U. L. P. of a corresponding short column; (2) on reasonable assumptions as to unintentional eccentricity; and (3) on the closely ascertained and well established modulus of elasticity. The best criterion of the utilizable capacity of a long column is sound analysis based on the maximum deflection consistent with good practice as well as on Conditions (2) and (3). Reasonable assumptions and simple methods are described and explained in Parts 5 and 6.

(*j*) For either combined direct compression, or tension and intentional flexure, reliance must be placed on sound analysis as there are only a few tests. These few tests, however, corroborate sound analysis.

(*k*) The ordinary theory of flexure is faulty, so that very short beams and girders should have considerably more section than such theory indicates, making greater provision for spreading the load and for ample web section and reinforcement at points of reactions and heavy concentrations of load. This is fully explained in Part 8.

(*l*) In beams and girders supported against lateral deflection and with flanges and webs of sufficient thickness to be amply stiff, the possibilities in the way of shifting the elastic zone and widening its limits under constant and fluctuating, but not alternating, loads, without undue deformation, are greater than for columns or tension members under direct stress.

(*m*) The U. L. P.'s of different pieces of steel of nominally the same grade, purchased under the same specifications, may differ greatly one from another. For instance, the U. L. P. of a short, 8-in., light **H**-column (web, 0.31 in., and flange, 0.40 in. nominal thickness) was 35 000 lb. per sq. in., whereas the U. L. P. of a short, 8-in., extra heavy **H**-column (web, 0.78 in., and flange, 1.212 in. nominal thickness) was only 19 000 lb. per sq. in.

(*n*) While great differences are possible in the U. L. P. of plates, shapes, and bars, and of members built from them, furnished under the same specifications, ordinarily they are all used with the same unit stress.

The paper concludes with the query whether it would not be well for manufacturers, fabricators, physical metallurgists, and structural engineers to investigate the practicability and desirability of grouping different sizes of plates, shapes, and bars of the same nominal grade so that the variation in the U. L. P. within each group would be comparatively small, and then proportioning structural members accordingly.

PART 1.—CONSTITUTION, ELASTICITY, DEFORMATION, ELASTIC FATIGUE, AND PERMANENT FATIGUE OF STEEL AND IRON

In general, metals are composed of irregular crystals, termed grains. Both steel and wrought iron are composed of sets or groups of crystalline grains. For either steel or wrought iron, separately considered, these sets have nearly the same modulus of elasticity but different elastic limits.

The deformation of metals is of two kinds: Elastic deformation, from which recovery is complete on release of stress; and plastic deformation, which is permanent. Elastic deformation, within the limitations of delicate measurements, is proportional to stress. Ewing, Rosenhain, and other eminent physical metallurgists have shown that plastic deformation of metals takes place within their grains by the sliding, one over another, of crystalline layers similarly as the cards within a pack slide while its form is being altered. Beilby has shown that thin films, between the sliding crystalline layers of the grains, acquire, by this mechanical movement, the mobility of the liquid state. While this mobility continues, the grains in which it occurs are in a state of elastic fatigue. This state is temporary and is followed either by recovery or permanent fatigue. After the stress that imparts mobility to thin films between crystalline layers of the grains is removed, the films resolidify, and either cement the adjacent layers, thus restoring to the grains their integrity and elasticity, or, if the over-straining has been too severe or long continued, leave the layers separated by cracks. The development of cracks constitutes permanent fatigue.

Short specimens of steel and iron, when tested in the usual way by increasing tension or compression, do not, as a rule, exhibit any plastic deformation discernible by ordinary inspection until a certain zone of comparatively small range is reached within which plastic deformation develops in such a large number of grains that a pronounced yield becomes plainly evident without the aid of a micrometer. The point at which the yield becomes plainly evident is known as the "yield point". In commercial practice, this point generally was, and still frequently is, erroneously termed the "elastic limit".

There are great differences in the recorded observations of the yield point of specimens from the same piece of steel in tests made under different conditions by different observers, as shown in Table 9; but for scientific purposes, a certain point in the yield (hereinafter designated) can be closely defined, and so well determined that it constitutes a reliable criterion.

PART 2.—ORIGINALLY IMPERFECT ELASTIC STATE OF SPECIMENS OF STEEL AND IRON; THE DEVELOPMENT, BY REPEATED CYCLES OF STRESS, OF FIELDS OF SEEMINGLY PERFECT ELASTICITY OF CHOSEN RANGE AND POSITION; AND THE LIMITATIONS TO SUCH DEVELOPMENT IMPOSED IN PRACTICE BY CRITICAL DEFORMATION

Measurements with delicate micrometers on long gauged lengths show that some slight plastic deformation occurs below, and even far below, the yield point in pieces of steel and iron when tested in the condition in which they

come from the rolls, forge, or mold. This indicates either that the elastic limits of pieces in such condition are very low or that there are no limits within which they are elastic. Experience and numerous tests show, however, that elastic limits bounding wide ranges of stress can be developed in pieces of steel and iron by repeatedly straining them through cycles of stress. Tests, especially those of Bairstow, also show that the elastic limits which can be developed in a piece of steel or iron are not constant quantities, characteristic of the piece, but are variable, depending on the conditions of straining.

Bairstow's experiments established what Bauschinger's tended strongly to show, that failure under cycles of stress repeated within certain limits is evidence that elastic limits have not been developed, and *vice versa*. This fact renders available, as a means of determining developed elastic limits and the laws governing their development, many published experiments in which failure or non-failure under cycles of stress from external forces has been recorded without stating the deformations.

The center of range of any cycle of stress from external forces is the mean between its extreme intensities. For instance, if the range is between a compression of 10 000 lb. per sq. in. and a tension of 20 000 lb. per sq. in., the center of range is a tension of 5 000 lb. per sq. in.

The maximum range of elastic deformation, with its opposite elastic limits, that can be developed by cycles of stress from external forces, varies with the position of the center of range, as shown in Fig. 1, in which, first, tension is measured upward; second, compression is measured downward; third, the loci of the elastic limits that bound the elastic range are shown for a typical piece of steel and indicate these limits for each elevation of the center of range above zero and each depression of the center of range below zero; and, fourth, each elevation and each depression of the center of range is indicated by an ordinate and an abscissa which are equal. For comparison, the critical deformation points and the maximum tensile strength, as determined in scientifically conducted tensile tests, are also shown in Fig. 1.

The writer has adopted as critical that point in the deformation below which it is predominantly elastic and above which the additional deformation is predominantly plastic. Plastic deformation is useful in adjusting structural members to the service to which they are subjected but, otherwise, the plastic state is objectionable *per se* while it lasts and the deformation remains an objectionable permanent set. Additional deformation predominantly plastic is seriously detrimental and in many cases is either dangerous or induces destruction under loads only slightly above those causing critical deformation, as discussed more in detail in Part 7. The point adopted as critical is the one designated by the Society's Special Committee on Steel Columns and Struts as the "Useful Limit Point" (U. L. P.).* If a member (including its details) is strained beyond its U. L. P. and survives the consequences of such temporary fatigue, its loss or impairment in usefulness is not, ordinarily, due to changes which took place in the structure of the steel, but to the lengthening or shortening, sometimes combined with bending, of the member and its details.

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–1920), p. 1618.

The elevation, even to very high limits, and the moderate depression of the elastic limits shown by the full lines in Fig. 1 are in direct accordance with experiments. The great depression indicated by the dotted lines is inferred, as search has failed to discover any direct experimental evidence.

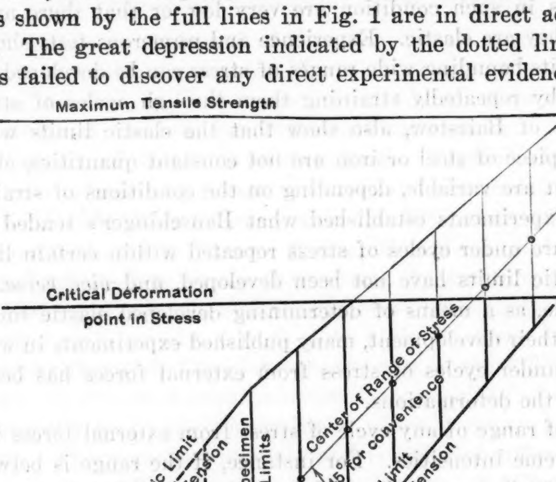

FIG. 1.—RANGE AND LIMITS TO SEEMINGLY PERFECT ELASTICITY DEVELOPED IN DIFFERENT SPECIMENS OF SAME PIECE OF STEEL BY REPEATEDLY STRAINING THEM BETWEEN LIMITS IN DIRECT TENSION, OR COMPRESSION, OR BOTH.

PART 3.—EXPLANATION OF THE PERFECTING OF ELASTICITY WITHIN LIMITS BY THE MODIFICATION OF INITIAL AUTO-STRESSES

As a basis for a satisfactory analysis and explanation of the development of elastic limits in a piece of steel or iron, it is necessary to consider the condition of the piece as regards auto-stresses and the changes that take place in these auto-stresses when the piece is over-strained and subsequently recovers from the resulting elastic fatigue.

Auto-stresses are the stresses within a body which, considered by themselves, without regard to those from external forces, form a balanced system of internal stresses. They may be present in a piece of steel or iron whether or not, at the same time, it is subjected to stresses from external forces. Auto-stresses are developed in a body while it is cooling from a hot state, and during recovery, or partial recovery, from the elastic fatigue of plastic deformation. They are also developed in structures during and by reason of such mechanical operations as tightening counter rods.

Auto-stresses which originate during the cooling of a body are caused by unequal rates or different periods of cooling of different parts. Every piece of steel or iron as it comes from the mill, forge, or foundry is liable to and probably does contain, in some of the grains of its weakest group, auto-stresses of such great intensity that the addition of even small stresses due to external forces will strain the metal beyond its elastic limit. A grain which has been over-strained may happen to recover from its elastic fatigue at such a point in the cycle of stress that the intensity of the total stress will be just the same as that of the stress from external forces, but the chances are greatly against such a phenomenon.

When the intensity of the total stress in a rehabilitated grain is different from the intensity of the stress from external forces, the difference is a new auto-stress. These new auto-stresses in rehabilitated grains are liable to be of any intensity between zero and the greatest possible, according to the point in the cycle at which recovery from elastic fatigue happens to take place. If this new auto-stress is less than a certain critical limit, and the intensities of the stresses from external forces under succedent cycles do not increase, the grain will not again be over-strained but will remain elastic; but if the new auto-stress is more than the certain critical limit the grain will again be over-strained during the next cycle, without, however, destroying the possibility that during the following or some later cycle it will recover from elastic fatigue and thereafter remain elastic. Under continued cycles of stress the fatigued grain will eventually either crack or recover with an auto-stress so low that the combined auto-stress and extreme stress from external forces during subsequent cycles will be within the elastic limits.

By reason of the changes in auto-stresses effected by elastic fatigue and recovery, if the range of stress in a given continuously repeated cycle is within certain limits successive adjustments of the auto-stresses in the various grains of a piece of steel or iron gradually take place through the operation of the law of chance, although in extreme cases a million or more cycles occur before the auto-stress in the last grain is adjusted, the plastic deformation in the meantime growing less and less until no plastic deformation or growth in permanent set under continued cycles of stress can be detected, even with delicate micrometers.

While the adjustment of auto-stresses and the cracking of some of the grains are taking place an improvement occurs in the elasticity of those grains which do not become cracked during recovery from elastic fatigue. This improvement is due to the fact that with the solidification of the thin mobile films new junctures, stronger than the original ones, are formed between the parts of the grains adjacent to these films.

From the foregoing analysis it appears that the results of repeated cycles of stress on pieces of steel and iron are the net sum of various and, to some extent, opposite effects. Analysis by process of reasoning is useful in explaining why and how changes take place, but the phenomena involved are far too complicated to admit of determining by mathematical analysis the limits to the range within which elasticity will be perfected by repeated cycles of

stress. Quantitative results of repeated cycles of stress can only be determined by experience and experiments.

Part 4.—Endurance Tests and Their Significance; and the Possibility of Remote Fatigue

Experiments on the endurance of specimens of steel and iron subjected to repeated cycles of direct stress have been made by Wöhler, Bauschinger, Reynolds and Smith, Bairstow, and the Sub-Committee of a Committee of the British Association for the Advancement of Science. The tests by the Sub-Committee were made by repeated tensions of specimens of two lots of steel with tensile strengths of 59 100 and 52 400 lb. per sq. in., respectively. These tests have not been published, but Mr. A. R. Fulton reported* "it was found that:

"1.—After ordinary yield is exceeded and permanent elongation has taken place, the limits of the proportionality of stress and strain are raised. The extent of this elongation for any maximum stress depends on whether the stress has been applied in one stage or with a sufficient number of repetitions at each of several stages.

"2.—The range of stress over which the proportionality is maintained is limited; once established it is independent of the number of cyclical repetitions to which it is subjected.

"3.—The range of stress may be again varied by further elongation, but to this there is a limit.

"4.—Failure under repetitions of tensile stress occurs by the movement of the crystals relatively to one another, consequent on the existing range of stress being exceeded, or during the transition period when one range is being changed to another.

"5.—Failure frequently originates by the range of stress, or even the maximum static stress of the material, being exceeded in parts of a section, and is due to mechanical flaws in the material or non-axial loading."

The fourth finding is open to the criticism that the movement occurs within the crystals. In addition to the tests mentioned, Thurston made endurance tests of steel and iron wires by subjecting them to long-continued constant tension. From these various experiments Tables 1, 2, 3, and 4 have been compiled. The data recorded, together with the supplementary data accompanying Figs. 2 and 3, and a few additional scattered citations, constitute an epitome of the important information to be gleaned from the published tests under direct stress.

Endurance tests by repeated cycles of transverse loading are comparatively numerous and embrace those of a recent and extensive investigation by the Engineering Experiment Station of the University of Illinois in co-operation with the National Research Council and others.† One set from the Watertown Arsenal Reports of ordinary hot rolled steel bars has been chosen for illustration (Table 5). In this set the only important variable in the fundamental conditions was a wide range of carbon in the steel. Auxiliary pulling tests were made. The experiments were well planned and

* Report, British Assoc. for the Advancement of Science, 1919, p. 484.

† *Bulletin No. 124*, Eng. Experiment Station, Univ. of Illinois.

scientifically conducted; and the number of cycles in each case was great and, in some cases, extreme.

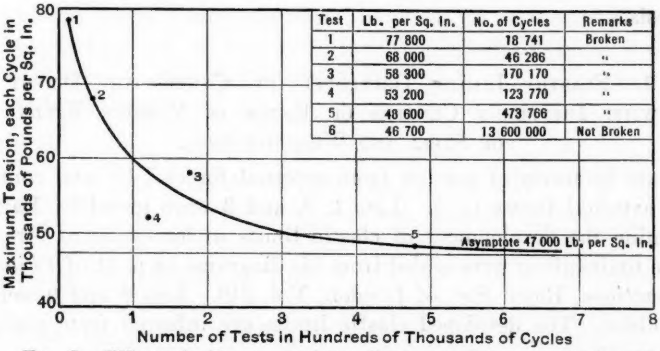

FIG. 2.—WÖHLER'S TESTS OF AXLE STEEL BY REPEATED TENSIONS FROM ZERO, SHOWING THE GRAPHIC DETERMINATION OF LIMITS TO THE CYCLE OF STRESS WHICH SEEMINGLY CAN BE ENDLESSLY REPEATED WITHOUT CAUSING FRACTURE. BY INFERENCE ELASTICITY IS PERFECTED WITHIN THESE LIMITS.

Bairstow made the tests recorded in Table 1 for Lots 1, 2, and 3, using the ordinary form of direct tension and compression testing machine with special attachments. His method was to test the specimen between the limits of stress selected for the experiment and to observe the deformation. In those cases in which the range between the limits was too great, the deformation progressively increased until the specimen broke. On the other hand, when the range was not too great, the increments of plastic deformation contributed by successive cycles became smaller and smaller so that eventually the deformation during each cycle seemed to be entirely elastic and no increase in permanent set under thousands of additional cycles could be detected with

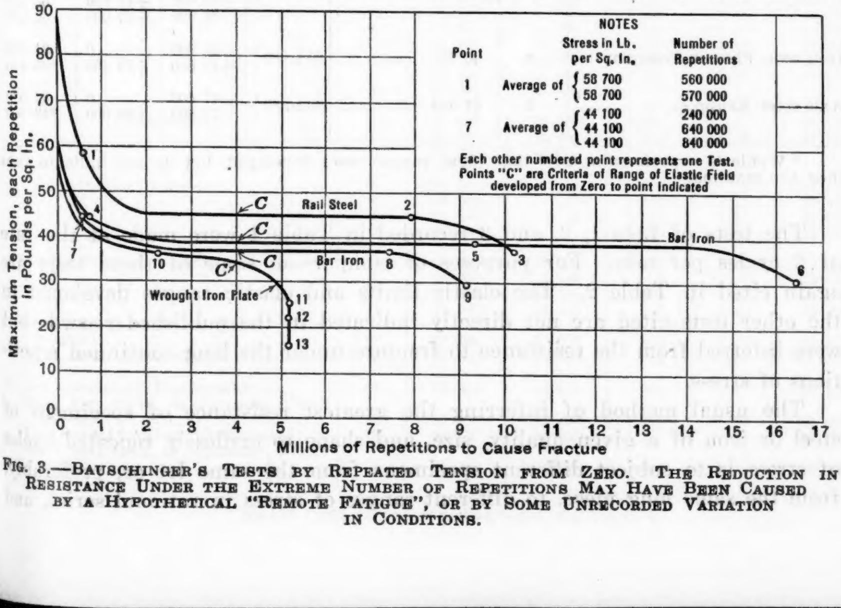

FIG. 3.—BAUSCHINGER'S TESTS BY REPEATED TENSION FROM ZERO. THE REDUCTION IN RESISTANCE UNDER THE EXTREME NUMBER OF REPETITIONS MAY HAVE BEEN CAUSED BY A HYPOTHETICAL "REMOTE FATIGUE", OR BY SOME UNRECORDED VARIATION IN CONDITIONS.

a measuring device sensitive to 0.00001 in. As long as the metal was to any appreciable extent plastic, the mobile films within the specimen acted as a drag on the elastic metal, delaying its recovery of shape, a phenomenon known as hysteresis.

TABLE 1.—Elastic Limits Developed by Cycles of Direct Stress with Different Centers of Range in Various Kinds of Steel and Wrought Iron.

(Stresses are in terms of tension from external forces (+) and compression from external forces (—). Lots 1, 2, and 3 were tested by Bairstow to determine the development of elastic limits under cycles of stress. The elastic limits given were scaled from his diagrams on p. 51 of *Philosophical Transactions,* Royal Soc. of London, Vol. 210. Lots 8 and 9 were tested by Wöhler. The developed elastic limits are inferred from stresses and endurance.)

Kind of materials.	Lot.	Tensile strength, in pounds per square inch.	Yield point, in pounds per square inch.	Developed Elastic Limits.			
				Cycles per minute.	Upper elastic limit.	Lower elastic limit.	Elastic range.
Unannealed charcoal iron......	1	43 900	32 500	2	+22 000 +40 000	−20 600 0	42 600 40 000
Unannealed axle steel...........	2	85 600	55 800	2	+30 000 +39 000 +51 000 +63 000 +74 000 +83 000	−28 000 −15 000 + 4 000 +18 000 +34 000 +45 000	58 000 54 000 47 000 45 000 40 000 38 000
Unannealed Bessemer steel......	3	99 900	71 700	2	+38 200 +58 000 +65 000 +83 000 +94 000	−35 700 −11 000 − 2 000 +18 000 +34 000	73 900 69 000 67 000 65 000 60 000
Iron axle, Phœnix Company....	8	46 700	....	60 to 80	+31 000 +42 800	0 +19 400	31 000 *23 400
Axle steel, Krupp's.............	9	94 500	....	60 to 80	+47 000 +77 800	0 +34 000	47 000 *43 800

* Wöhler's experiments show that these ranges were developed, but do not indicate that they are maxima.

The tests of Lots 1, 2, and 3 recorded in Table 1 were made at the rate of 2 cycles per min. For purposes of comparison some of these tests are again cited in Table 2. The elastic limits and elastic ranges developed in the other tests cited are not directly indicated in the published records but were inferred from the resistance to fracture under the long continued repetitions of stress.

The usual method of inferring the greatest resistance of specimens of steel or iron of a given quality, size, and shape to endlessly repeated cycles of stress is to subject different specimens from the same lot or, preferably, from the same long piece, to different ranges of stress in rational series, and

to repeat each test until failure occurs or circumstances indicate that the test might be repeated indefinitely without breaking the specimen. The plotted results of a series of such tests indicate by an asymptote the maximum range within which seemingly the stress can be endlessly repeated without fracture; and within which, by inference, seemingly perfect elasticity has been developed. This is illustrated in Fig. 2. Such tests are commonly called tests of fatigue.

TABLE 2.—RANGES OF STRESS BETWEEN ELASTIC LIMITS DEVELOPED BY CYCLES OF ALTERNATE DIRECT STRESSES.

(Lots 1, 2, 3, 7, 10, 11, 12, 13, 14, 15, 16, and 17 were tested by Bairstow (See *Philosophical Transactions,* Royal Soc. of London, Vol. 210, and *Minutes of Proceedings,* Inst. C. E., Vol. CLXVI, 1906, Pt. IV). Lots 4, 5, and 6 were tested by Reynolds and Smith (See *Philosophical Transactions,* Royal Soc. of London, Vol. 199, pp. 265-297).)

MATERIAL.		ORDINARY SPECIMEN TENSILE TESTS.		ELASTIC RANGE DEVELOPED BETWEEN TENSION, *T*, AND COMPRESSION, *C*.		
Kind.	Lot.	Maximum strength, in pounds per square inch.	Yield point, in pounds per square inch.	Cycles per minute.	Ratio of *T* to *C*.	Range, in pounds, per square inch.
Charcoal iron (Unannealed)..........	1	43 900	32 500	1 200	1.07	40 800
" " "	1	43 900	32 500	800	1.4	42 600
" " "	1	43 900	32 500	2	1.07	*42 600
Lowmoor iron (Annealed)............	5	51 700	......	1 900	1.14	26 500
" " "	5	51 700	......	1 600	1.17	39 200
" " (Unannealed)..........	5	52 800	36 800	....	....	......
Wrought iron bar (Unannealed).....	16	53 200	36 900	800	1.4	46 000
" " " "	17	57 300	32 900	800	1.4	43 000
Mild steel (Annealed)................	4	57 400	41 800	1 930	1.13	27 700
" " "	4	57 400	41 800	1 750	1.14	34 000
" " "	4	57 400	41 800	1 670	1.12	40 800
" " "	4	57 400	41 800	1 500	1.15	43 200
" " "	4	57 400	41 800	1 400	1.16	45 000
" " "	4	57 400	41 800	1 325	1.17	46 600
" " "	4	57 400	41 800	....	....	......
" " (Unannealed)..............	4	55 000	38 300	2 030	1.13	29 100
" " "	4	55 000	38 300	1 305	1.18	47 300
" " "	4	55 000	38 300	....	....	......
Cast steel (Annealed)................	6	107 500	......	1 900	1.13	27 100
" " "	6	107 500	......	1 750	1.15	37 400
" " "	6	107 500	......	1 660	1.16	41 000
" " "	6	107 500	......	1 300	1.18	45 000
Bessemer steel (Unannealed)..........	3	99 900	71 700	1 200	1.07	59 400
" " "	3	99 900	71 700	2	1.07	*73 900
" " "	10	106 600	65 800	800	1.40	70 000
" " "	11	98 000	62 800	800	1.40	66 000
Steel bar, ⅞ in. in diameter (Unannealed)..............	12	98 200	50 000	800	1.40	63 400
Axle steel (Unannealed)..............	2	85 600	55 800	2	1.07	*58 000
Bessemer steel (Unannealed)..........	13	63 900	53 300	800	1.40	58 800
Steel bar, 2½ in. in diameter (Unannealed)..............	14	63 400	35 400	800	1.40	57 000
Large steel forging (Unannealed)....	15	66 000	32 500	800	1.40	46 100
Steel bar, ⅞ in. in diameter (Unannealed)..............	7	49 100	30 100	800	1.40	42 400

* Scaled from Bairstow's diagrams, see Table 1.

TABLE 3.—Resistance to Fracture Under Repeated Cycles of Direct Stress.

(Compiled from experiments by Wohler, Bauschinger, Reynolds and Smith, and Bairstow. All stresses are given in units of 1 000 lb. per sq. in.)

	Tests made by	Specimens from	Carbon, percentage.	Maximum tensile strength.	Primitive elastic limit.	Resistance to Fracture, 4 000 000 Cycles of Stress.			Extreme Tests.					
						From tension.	To tension.	Range.	From tension.	To tension.	Range.	Number of cycles, in millions.	Frequency per minute.	Results.
1	Wohler.	Iron axle		46.7		0	31.0	31.0	0	31.1	31.1	10.1	70 about.	Broken.
2		" "		46.7		23.3	42.8	19.5	19.4	42.8	23.4	2.4	"	Not broken.
3		Axle steel		94.5		0	47.0	47.0	0	46.7	46.7	13.6	"	Not broken.
4		" "		94.5		34.0	77.8	43.8	34.0	77.8	43.8	12.1	"	Not broken.
5		Cast iron				0	9.7	9.7	0	9.7	9.7	7.6	"	Broken.
6		" "							0	9.7	9.7	0.2	"	Broken.
7	Bauschinger.	Mild steel plates		63.8	34.9	0	35.8	35.8	0	35.8	35.8	11.0		Not broken.
8		" " "		63.8	34.9				0	35.8	35.8	3.5		Broken.
9		Mild steel boiler-plates		59.6	39.4	0	39.0	39.0	0	36.7	36.7	6.5		Not broken.
10		" " " "		59.6	39.4				0	41.9	41.9	4.9		Not broken.
11		" " " "		59.6	39.4				0	36.7	36.7	0.3		Broken.
12		Steel axle		89.8	39.4	0	44.1	44.1	0	44.1	44.1	9.0		Broken.
13		Rail steel		87.4	42.6	0	44.1	44.1	0	36.7	36.7	10.2		Broken.
14		Bar iron		59.8	33.2	0	38.5	38.5	0	30.9	30.9	16.5		Broken.
15		" "		59.6	26.4	0	36.7	36.7	0	29.6	29.6	9.1		Broken.
16		Wrought-iron plate		56.4	15.2	0	34.0	34.0	0	15.9	15.9	5.2		Broken.

TABLE 3.—(*Continued.*)

	Tests made by	Specimens from	Carbon, percentage.	Maximum tensile strength.	Primitive elastic limit.	Resistance to Fracture, 1 000 000 Cycles of Stress.				Extreme Tests.					
						From compression.	To tension.	Range.	Ratio, tension to compression.	From compression.	To tension.	Range.	Number of cycles, in millions.	Frequency per minute.	Results.
	Reynolds and Smith.	MILD STEEL BARS:													
17		Unannealed	....	55.0	....	22.2	25.1	47.3	1.18	13.3	15.1	28.4	2.0	2 015	Not broken.
18		"	"	55.0	"	13.3	15.8	29.1	1.18	21.6	25.6	47.2	1.1	1 305	Broken.
19		Annealed	....	57.8	....	13.0	14.7	27.7	1.18	12.3	14.0	26.3	1.8	1 888	Not broken.
20		"	"	57.8	"	15.9	18.1	34.0	1.14	15.0	17.1	32.1	5.1	1 696	Not broken.
21		"	"	57.8	"	19.4	21.7	40.8	1.12	19.4	22.1	40.5	0.7	1 682	Broken.
22		"	"	57.8	"	20.1	23.1	43.2	1.16	20.7	23.9	44.6	0.5	1 544	Broken.
23		"	"	57.8	"	20.8	24.2	45.0	1.16	21.2	24.7	45.9	0.7	1 441	Broken.
24		"	"	57.8	"	21.5	25.1	46.6	1.17	21.8	25.6	47.4	0.7	1 345	Broken.
		CAST STEEL:													
25		Annealed	....	107.5	....	12.7	14.4	27.1	1.13	12.4	14.1	26.5	1.3	1 892	Broken.
26		"	"	107.5	"	17.4	20.0	37.4	1.15	17.9	20.5	38.4	0.7	1 838	Broken.
27		"	"	107.5	"	19.0	22.0	41.0	1.16	18.8	21.8	40.6	2.3	1 650	Broken.
28		"	"	107.5	"	20.7	24.5	45.2	1.18	20.2	23.8	44.0	0.9	1 303	Broken.
		LOWMOOR IRON:													
29		Annealed	....	51.7	....	12.4	14.1	26.5	1.14	12.3	13.9	26.2	1.2	1 890	Broken.
30		"	"	51.7	"	18.1	21.1	39.2	1.17	18.2	21.3	39.5	0.8	1 630	Broken.
31		Cast iron	....	....	....	....	....	....	....	5.9	6.9	12.8	0.6	1 305	Broken.
	Bairstow.	STEEL:													
32		Bessemer	0.645	106.6	62.0	29.4	41.2	70.6	1.4	29.2	41.0	70.2	1.6	795	Not broken.
33		"	0.446	98.0	56.0	28.4	39.7	68.1	1.4	27.2	38.0	65.2	1.8	796	Not broken.
34		"	0.170	63.9	48.0	25.0	35.0	60.0	1.4	24.0	33.5	57.5	2.1	795	Not broken.
35		Bar, 7/8 in. in diameter.	0.446	98.2	43.9	26.4	37.0	63.4	1.4	26.0	36.5	62.5	3.4	796	Not broken.
36		" 2½ in. in diameter.	0.331	63.4	32.0	24.1	33.7	57.8	1.4	21.7	30.5	52.2	2.0	794	Not broken.
37		" 7/8 in. in diameter.	0.065	49.1	24.0	17.7	24.7	42.4	1.4	17.7	24.7	42.4	1.1	791	Broken.
38		Large forging	0.336	66.0	29.0	19.2	26.9	46.1	1.4	17.8	25.0	42.8	1.8	794	Not broken.
39		Charcoal iron bar	0.039	43.9	29.0	17.8	24.8	42.6	1.4	17.3	24.1	41.4	1.4	792	Not broken.
40		" " "	0.039	43.9	29.0	20.5	22.3	42.8	1.09	20.2	22.1	42.3	1.3	818	Not broken.
41		" " "	0.039	43.9	29.0	21.9	20.2	42.1	0.92	20.6	18.9	39.5	1.0	800	Not broken.
42		" " "	0.039	43.9	29.0	23.8	17.2	41.0	0.72	24.1	17.3	41.4	1.5	800	Not broken.
43		" " "	0.039	43.9	29.0	18.6	19.9	38.5	1.07	18.6	19.9	38.5	1.0	1 200	Not broken.
44		Wrought-iron bar	0.029	53.2	32.0	20.2	28.2	48.4	1.4	18.9	26.4	45.3	1.1	797	Not broken.
45		" " "	0.195	57.3	29.9	17.9	25.1	43.0	1.4	18.0	25.2	43.2	0.9	798	Broken.
46		" " "	0.195	57.3	29.9	20.5	22.7	43.5	1.09	20.7	22.6	43.3	1.0	818	Not broken.
47		" " "	0.195	57.3	29.9	22.7	20.8	43.5	0.92	25.2	18.1	43.3	1.0	800	Not broken.

TABLE 4.—THURSTON'S TESTS OF ENDURANCE OF IRON UNDER DEAD LOADS.*

Percentage of maximum static load.	TIME UNDER LOAD BEFORE FRACTURE.	
	Hard unannealed wire.	Soft unannealed wire.
95	8 days	3 min.
90	35 days	5 min.
85	Unnoted, but 1 or 2 years	261 days
80	91 days	266 days
75	Unbroken after several years	17 days
70	Same results	455 days
65	Same results	455 days (probable jar)

* Thurston's Table No. 11, *Transactions*, Am. Soc. C. E., Vol. XLI (1899), p. 516. Thurston states that "some of these wires were still unbroken in 1898, after 15 years loading." His records do not mention deformation, but, from the known relation between endurance and elasticity, it can be inferred that each specimen which did not break had developed an elastic limit at and under the load endured.

TABLE 5.—ENDURANCE TESTS OF ROTATING SHAFTS. MATERIAL, HOT-ROLLED BARS OF OPEN-HEARTH STEEL TURNED TO 1.009 INCHES IN DIAMETER, FROM 1.25 INCHES.

(Length between supports, 33 in.; loaded over 4 in. length at middle; speed of rotation, 500 rev. per min. Compiled from Watertown Arsenal Reports for 1903-1908.)

PERCENTAGES OF CHEMICAL CONSTITUENTS.			PULLING TESTS AUXILIARY TO ENDURANCE TESTS. GAUGED LENGTH, 4 IN.		ENDURANCE TESTS OF ROTATING SHAFTS.			
					Developed elastic limit or one-half range, in pounds per square inch.	Extreme Tests.		
Carbon.	Manganese.	Silicon.	Maximum strength, in pounds per square inch.	U. L. P.,* in pounds per square inch.		Extreme fiber stress, in pounds per square inch.	Number of rotations.	Result of test on shaft.
0.17	0.57	0.04	68 000	†51 000	35 000	30 000	100 000 000	Not broken.
0.34	0.65	0.34	84 600	54 300	40 000	40 000	63 667 320	Broken.‡
0.55	0.75	0.14	106 100	57 000	35 000	35 000	75 006 000	Not broken.
0.73	0.64	0.04	140 700	64 500	45 000	40 000	58 400 000	Not broken.
0.82	0.36	0.10	142 250	63 000	44 000	40 000	202 000 000	Not broken.
1.09	0.39	0.11	182 800	76 200	38 000	35 000	175 280 000	Not broken.

* In each case, the U. L. P. was obtained from deformation given in the reports of the Watertown Arsenal, but the term does not appear therein.

† Load fell to 42 000 lb. per sq. in. which is 61.8% of the maximum strength.

‡ Bar ruptured midway between bearings, at a score mark made by the head of the screw which holds up the middle bearing fixture.

Besides elastic fatigue and permanent fatigue, steel and iron may be liable to what may be called, for want of a better name, remote fatigue. Every change in the temperature or stress to which a specimen of steel or iron is subjected affects it, perhaps permanently as regards electricity and magnetism. Moreover, some permanent fatigue may be developed at such a slow rate, and be so localized and, for a long time, confined within such narrow limits, that it gives no appreciable indication of its presence in endurance

tests carried to what seems to be a sufficient number of cycles. Slow changes are continually taking place in all material things. Even atoms disintegrate, electrons are radiated into space, and planets, suns, and universes come into being, exist for a while, and then pass away. It is not reasonable to expect metals to endure forever no matter how moderate the strain nor how great the protection against corrosion and abrasion.

Bauschinger's experiments on rail steel, bar iron, and wrought-iron plate, cited in Table 3, and in Fig. 3, seem to show that failure from remote fatigue is a reasonable possibility even after seemingly perfect elasticity has been developed within certain limits. The results tabulated may be due, however, to differences in quality of the specimens or, possibly, to unrecorded occurrences during the tests; or it may be that there are some other unexplained circumstances by reason of which these tests are not proper criteria. Reciprocating parts of locomotives, motor vehicles, and stationary engines are sometimes subjected in service to repetitions of stress much greater in number than those noted in Table 3.

Endurance tests by bending stresses are much more numerous than by direct stresses; and the extreme tests are usually carried to a much larger number of cycles, the largest of which the writer has knowledge being 202 000 000, as shown in Table 5. Among the bending tests there are very few failures after elasticity has been seemingly perfected and these few can generally be accounted for by scratches, set screws marks, or other causes incident to testing; in other words, bending tests do not strongly indicate "remote fatigue". Endurance tests by direct stress carried to 50 000 000 cycles or more are much to be desired to disprove or confirm the indications of the extreme tests made by Bauschinger (Fig. 3).

In dealing with machinery, the possibility should be seriously considered of failure from "remote fatigue" after elasticity has been seemingly perfected and no appreciable increase in permanent set has been developed thereafter under millions of cycles of stress; but few if any structures, or parts thereof, will be strained often enough to fail from "remote fatigue" before they fail from other causes or are put out of service. The possibility, if there is a possibility, of the failure of a structure from "remote fatigue" is not an element to be considered in its design, but rather in deciding whether or not an old structure should be condemned.

The development of upper and lower elastic limits, both in tension, is, perhaps, the most astonishing result achieved by straining pieces of ordinary iron and steel by cycles of stress. The occurrence of this phenomenon indicated that a large proportion of the grains either had at the start, or acquired during the test, elastic limits in tension considerably greater than the range of stress between the upper and lower elastic limits of the piece. The stress in the grains that determined the lower elastic limit of the piece is the net result of an "auto-compression," developed in them during the test, and tension from external forces. This auto-compression is so great that when the minimum tension from external forces is reached at the lower elastic limit of the piece the auto-compression in the critical grains exceeds the tension

from external forces by an amount equal to the elastic limit of these grains in compression.

It may be inferred that a piece of ordinary iron or steel can have its elastic field lowered by cycles of direct loading to such an extent that both the upper and lower elastic limits will be in compression; but tests to substantiate this are lacking. As shown in Table 2 experiments on pieces of iron and steel under cycles of direct stress indicate that a variation in the rate of from 2 to 800 cycles per min. makes little difference in the range of the elastic field that can be developed. They also indicate that the range is reduced with an increase in the rate above 800 cycles per min.

Some experiments with intervals of hours, days, or even years between successive cycles indicate the development of much greater ranges of elastic stress than with rates of from 2 to 800 cycles per min.; but because of the small number of cycles in these experiments the accumulation of defects and permanent sets may be so minute as to be undetectable with the micrometers used.

Experiments indicate that the elastic limit can be increased by allowing a specimen to remain at rest for a considerable period under strain, provided the strain is not too great. Thurston's experiments in this regard are shown in Table 4. His records do not mention deformation; but, from the known relations between endurance and elasticity it can be inferred that each specimen which did not break had developed an elastic limit corresponding to the load.

PART 5.—INTERPRETATION OF TESTS AS CRITERIA OF UTILIZABLE CAPACITY OF MEMBERS IN DIRECT TENSION OR COMPRESSION

For members subjected to direct tension only, or if sufficiently stiff, to direct compression only, the criterion of utilizable capacity is either the elastic limit that can be developed or the point of critical deformation, whichever is the lower; and the ratio of this criterion to the computed direct stress in service, is the factor of safety. Unfortunately, this basic criterion can only be determined directly and accurately by endurance tests.

In practice, the utilizable capacity has to be inferred from the results of simple tensile or compressive tests under gradually increasing loads. Some of these results, such as the elongation and reduction in area, are indications of quality. The ultimate tensile strength and the development of the yield, if the steel is of suitable quality, are fairly reliable criteria of utilizable capacity when jointly considered as hereinafter described.

When, in a tension or fairly rigid compression member under a constant load, any part of the metal is strained beyond its elastic limit by the stress from the load combined with auto-stress, plastic deformation will occur and will continue for a time; but, unless the load is too near the maximum, it will eventually cease and become a permanent set. Slight permanent sets in structural members are unavoidable; large permanent sets are ruinous. Between these conditions are sets which are seriously detrimental without being immediately ruinous. Plastic deformation (which is the original form in which the extension or linear compression constituting a set exists) after it becomes

serious, because of the alterations it causes in the lengths of structural members, generally increases with the stress much more rapidly in soft, than in medium, and in medium, than in hard, steel; hence after serious plastic deformation has begun, there is much greater liability to the development of ruinous deformation in soft than in hard steel without, in some cases, any increase in load.

It is desirable for purposes of analysis and of definiteness in stating requirements to define precisely a readily ascertainable point in tests which will be regarded as the beginning of serious plastic deformation. In formulating such a rule close approximation to accuracy is the primary consideration and ease of ascertainment secondary.

In the Final Report of the Society's Special Committee on Steel Columns and Struts, it is stated*:

"In the Progress Report of 1917, the Committee mentioned the discussion held by the American Society for Testing Materials at its Annual Meeting, June, 1916, on the relation between proportional limit, elastic limit, and yield point. For the purpose of studying the column tests, the Committee gave careful consideration to this discussion to find whether it was possible to determine some point which, for practical purposes, might be easily located, clearly defined, and at the same time represent the limit where the metal ceases to have structural value. None of the terms defined by the discussions of the American Society for Testing Materials has appealed to the Committee as having these qualities. In searching for a more satisfactory definition, the Committee considered a modification of the suggestion made some years ago by the late J. B. Johnston, M. Am. Soc. C. E. The Committee has defined the critical point as the point which is determined graphically by drawing a line tangent to the envelope of the stress-strain curve, having a slope of one-half that of the last run-up line for its straight, or nearly straight, portion. Fig. 7 [of the Committee's report] illustrates this method of determination. So as not to confuse this with former definitions of yield point or elastic limit, the Committee has adopted a new term, and calls this the Useful Limit Point, or U. L. P.

"* * * It will be noted that the method is applicable to both tension and compression tests."

The normal U. L. P. is the same in compression as in tension for all steel likely to be used in structural members. This is shown in Table 6. It was also shown exhaustively and conclusively by the late Charles A. Marshall, M. Am. Soc. C. E., in his paper entitled "Compressive Strength of Steel and Iron".†

The Special Committee on Steel Columns and Struts precisely defined the most useful point that has yet been selected as the demarcation between characteristically elastic and characteristically plastic deformation. This was in terms of graphics; defined without recourse to graphics, the U. L. P. is that point at which under gradually and slowly increasing stress the rate of combined plastic and elastic deformation becomes twice the rate of elastic deformation. For instance, if the increase in deformation during the yield is measured for each additional 1 000 lb. per sq. in. and the modulus of elasticity, E, is 30 000 000

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1618.
† *Transactions*, Am. Soc. C. E., Vol. XVII (1887), p. 53.

lb. per sq. in., the U. L. P. will be reached during the first increase of 1 000 lb. per sq. in. in which the increase in deformation reaches $\frac{\text{Gauged length}}{15\,000}$.

TABLE 6.—COMPARATIVE TESTS IN TENSION AND COMPRESSION OF OPEN-HEARTH STEEL BARS.

(All specimens turned to 1.0092 in. in diameter from 1.25 in. in diameter; tension specimens, gauged length, 30 in.; compression specimens, 12 in. long, with flat ends and a ratio, $\frac{l}{r}$, of 47.6. Tests made at the Watertown Arsenal and given in reports for 1886, pp. 1635-1653; and for 1887, pp. 802-822.)

Marks.	Composition.			Tension, in Pounds per Square Inch.			Compression, in Pounds per Square Inch.		
	Carbon, percentage.	Manganese, percentage.	Silicon, percentage.	Maximum strength.	U. L. P.*	Modulus of elasticity.	Maximum strength.	U. L. P.*	Modulus of elasticity.
833	0.09	0.11	....	52 475	30 000	30 151 000	32 125	30 500	30 120 000
123	0.20	0.45	....	68 375	39 500	30 151 000	39 190	37 000	30 303 000
782	0.31	0.57	....	80 600	46 500	30 000 000	45 500	44 500	30 612 200
795	0.37	0.70	....	85 160	50 000	30 151 000	50 875	47 000	31 250 000
803	0.51	0.58	0.02	98 700	58 000	30 000 000	58 000	57 000	30 075 000
797	0.57	0.93	0.07	117 440	55 000	30 104 000	65 500	55 500	30 201 000
823	0.71	0.58	0.08	116 750	55 000	30 088 000	65 440	55 500	31 034 000
750	0.81	0.56	0.17	149 600	70 000	29 923 000	87 750	74 500	30 000 000
756	0.89	0.57	0.19	141 290	78 000	29 864 000	84 125	77 000	30 612 000
334	0.97	0.80	0.28	152 550	80 000	29 817 000	91 500	83 000	30 822 000

NOTE.—The modulus of elasticity of a nickel steel, flat-ended column with a ratio, $\frac{l}{r}$, of 100, a resistance to failure of 48 910 lb. per sq. in., and a U. L. P. of 47 000 lb. per sq. in., computed from compression sets measured at four points, in a gauged length of 50 in., was 29 800 000 lb. per sq. in. The test is reported in Watertown Arsenal Report, 1910, pp. 176-177. The moduli of elasticity, similarly computed except that the gauged length was 80 in., from column tests reported in *Technologic Paper 101*, U. S. Bureau of Standards, are as follows: (p. 48) chrome steel, 29 100 000; (p. 36) silicon steel, 30 400 000; (p. 46) silicon steel, 29 500 000; (p. 32) Mayari steel, 29 000 000; and (p. 42) Mayari steel, 30 300 000. For the chemical analyses, see Table 18.

* In each case, the U. L. P. was obtained from deformation given in the reports of the Watertown Arsenal, but the term does not appear therein.

Precise values of E cannot be observed in a piece of steel after it has been strained to its U. L. P. until the films of metal made mobile by the strain have resolidified. The value of E for steel of all carbon contents up to and including 0.97% is approximately 30 000 000 as shown in Table 6 and, therefore, it is hardly worth while obtaining values of E (which may be only approximate) for each piece by "run-ups". The value indicated by the last run-up in the columns for which the Special Committee on Steel Columns and Struts submitted graphs varies from 28 000 000 to 30 000 000, with an average of 29 139 000.

The acceptance of the U. L. P. as a criterion of utilizable capacity without serious plastic deformation is qualified by four considerations: First, the rate at which the test is conducted must not be excessive; second, when the U. L. P is greater than the limit to which elasticity can be seemingly perfected by

cycles of stress, a suitable deduction should be made in determining the utilizable capacity; third, allowance should be made for the fact that, in testing specimens by gradually increasing the load, the load is liable to drop to a point below the U. L. P. immediately after a yield point is reached; and, fourth, all the U. L. P.'s of the full-sized columns tested by the Column Committee were lower, as far as data were available for comparison, than the weighted average of the U. L. P.'s of the specimen tests.

No attempt is here made to define what constitutes an excessive rate of speed in testing for the purpose of determining the maximum strength and the U. L. P. Although published information on the subject is scarce, a search has disclosed the material in Tables 7, 8, and 9. It is reasonably certain that none of the Watertown Arsenal's or of the U. S. Bureau of Standard's tests cited, was made at an excessive rate of speed.

TABLE 7.—Tests of Wrought Iron to Show the Effect of Slow and Rapid Fractures on Specimens of Varying Length.

(Specimens were taken from a bar of iron 1⅜ in. square, reduced to a diameter of 1.008 in. (area, 0.08 sq. in.). From Watertown Arsenal Report for 1887, p. 924.)

Length of reduced section, in inches.	Tensile strength, in pounds per square inch.	Duration of test.	Gauged length, in inches.	Percentage of elongation in gauged length.	Percentage of contraction.	Area at rupture, in square inches.
0.08	49 380	6 min.	....	....	49.1	0.407
0.08	50 750	8 sec.	....	....	37.1	0.503
1.60	47 730	10 min.	1	56.0	47.6	0.419
1.60	49 130	13 sec.	1	50.0	46.2	0.430
2.40	47 980	8 min.	2	39.0	43.2	0.454
2.40	49 000	14 sec.	2	41.5	49.1	0.407
3.20	47 070	10 min.	3	39.0	49.1	0.407
3.20	48 120	15 sec.	3	36.0	47.6	0.419
4.80	46 860	10 min.	4	32.0	43.2	0.454
4.80	48 000	18 sec.	4	32.8	47.6	0.419
6.40	47 450	9 min.	6	25.0	49.1	0.407
6.40	48 250	20 sec.	6	32.0	47.6	0.419
8.00	45 650	9 min.	8	25.9	41.7	0.466
8.00	46 000	30 sec.	8	25.9	41.7	0.466
Mean of slow tests..	47 446	8.9 min.	....	36.1	46.1	0.431
Mean of rapid tests..	48 464	16.9 sec.	....	36.4	45.3	0.438
Grooved............	57 200	6 min.	....	....	32.4	0.541
Grooved............	59 250	6 sec.	....	....	32.4	0.541

The Special Committee on Steel Columns and Struts states*:

"The ultimate strengths shown on the supplementary specimen tests are very close to the ultimate strengths given for the specimen mill tests, and neither the ultimate strengths from the supplementary tests nor from the mill tests indicate the falling off in strength of the thicker material. It is also evident that the

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919–20), p. 1612.

yield point, as recorded by the ordinary commercial tensile specimen tests, even when the machine is run at comparatively slow speeds, as was done in the Pittsburgh Laboratory of the Bureau of Standards, does not give the correct index of the strength of the material."

In this regard a report of the Committee on Iron and Steel Structures of the American Railway Engineering Association, of tests on columns by the U. S. Bureau of Standards, states:* "The specimen tensile tests on which material is ordered and accepted afford no proper criterion for the strength of a column."

TABLE 8.—EFFECT OF VARIATIONS IN SPEED OF TESTING ON THE PHYSICAL PROPERTIES OF STEEL RIVET RODS.

(a)* TESTS MADE BY THE PENNSYLVANIA STEEL COMPANY ON TEN RIVET RODS TAKEN FROM AN ACID OPEN-HEARTH HEAT. FROM EACH ROD FIVE BARS WERE CUT AND EACH BAR WAS BROKEN AT A DIFFERENT SPEED.

Physical properties.		Speed of Testing Machine, 1 Inch in:				
		13.3 sec.	20 sec.	1.5 min.	2.6 min.	14.3 min.
Tensile strength, in pounds per square inch	Maximum....	61 870	61 580	60 640	60 240	59 760
	Minimum....	60 200	59 580	58 760	58 240	58 200
	Average of 10	61 075	60 672	59 523	59 387	59 027
Yield point by drop of beam, in pounds per square inch	Maximum....	46 920	46 300	44 810	43 430	40 480
	Minimum....	44 360	43 400	41 690	40 810	38 950
	Average of 10	45 708	44 410	42 904	41 763	39 647
Elongation in 8 in., percentage	Maximum....	32.00	32.00	31.00	31.00	34.00
	Minimum....	27.75	27.00	27.50	28.00	30.25
	Average of 10	29.65	30.18	29.45	29.33	31.93
Reduction of area, percentage	Maximum....	67.1	66.3	68.0	67.1	68.4
	Minimum....	62.3	62.4	63.9	63.1	63.4
	Average of 10	65.0	64.8	65.8	65.7	66.5

(b)† TESTS MADE BY MR. E. A. CUSTER FOR THE BALDWIN LOCOMOTIVE WORKS.

Number of tests.	Pulling speed, 1 in. in:	Tensile strength, in pounds per square inch.	Field point by drop of beam, in pounds per square inch.
6	3 min.	56 820	37 510
8	15 sec.	58 870	40 530

* These data were taken from "The Manufacture and Properties of Iron and Steel", by Harry Huse Campbell, Gen. Mgr., The Pennsylvania Steel Co., 1903, p. 453.

† *Loc. cit.*, p. 457.

From Tables 1 and 2 it appears that under alternate direct tension and compression the limits to the range within which elasticity both in tension and compression can be perfected are lower than the yield point and probably lower in all cases than the direct tension and compression U. L. P.'s. It is probably the uncombined soft alpha iron in steel that determines the limits to the range of developed elastic fields. This fact tends to limit the range in steel that has not been cold-drawn or cold-rolled, or heated and quenched,† to what it is in

* *Proceedings*, Am. Ry. Eng. Assoc., 1918, p. 793.

† Steel which has been cold-drawn or cold-rolled or heated and quenched, is outside the scope of this paper.

wrought iron or in soft steel (sometimes called ingot iron). The opportunity under direct stress for increasing the elastic range of iron by resolidification of films made mobile by strain is, however, greater in the iron of high carbon steel than in that of medium carbon steel, likewise, greater in the iron of medium carbon steel than in that of soft low carbon steel or wrought iron. On the whole, the advantage is with soft steel and wrought iron over medium and high carbon steel as regards the ratio of the elastic range to the U. L. P. that can be developed by alternate direct tension and compression both of which are within the U. L. P. There is no reasonable chance of developing in service resistance to repeated direct stresses much above the U. L. P. in compression or tension members of structures (except possibly in adjustable rods of Howe trusses), on account of the serious, and in some cases, ruinous deformation that would occur under such stresses. After Wöhler had shown that endurance limits in test pieces can be developed much above the yield point and, therefore, much above the U. L. P., Launhardt published a formula designed to show this endurance limit for different ratios of maximum to minimum stresses, both in tension or both in compression; but, with the possible exception of adjustable rods in Howe trusses, Launhardt's formula has no rational applicability to tension and compression members of structures and machines. The occasion for referring to it is the fact that William Cain, M. Am. Soc. C. E., modified Launhardt's formula empirically "to allow for all the influences of impact", introduced a factor of safety, and recommended the formula thus modified for the design of railroad bridge members. This was forty-eight years ago,* but the modified formula is still extensively used.

As a criterion of utilizable capacity, the U. L. P.'s from specimen tests are too high rather than too low even for direct stress that is static or fluctuating but never reversed. In an extreme case (Test 7934, Table 10), soft steel with a maximum tensile strength of 48 240 lb. per sq. in. seemed in a careful pulling test to be perfectly elastic up to 40 000 lb. per sq. in. and had a U. L. P. of 40 600 lb. per sq. in., or 84.2% of the tensile strength. This U. L. P. is far above any valuation of utilizable capacity justified by endurance tests. Valuable information on this subject was obtained in Bairstow's experiments on an axle steel, with a yield point of 24.9 tons per sq. in., which in repeated loadings from 0 to 23.2 tons per sq. in. showed no lack of elasticity until after 6 000 loadings, when a permanent extension of the order of the yield took place.†

Marshall‡ gives a number of test diagrams in which the elastic limit, as he calls it, is quite close to the U. L. P., and states:

"*First.*—When a load equal to the elastic limit of any of these materials is imposed and allowed to remain for some time, it causes permanent change of length, amounting, it is believed, to much more than has been generally understood, and which is apparently a very definite quantity for each case. *Second.*—This change of length, having begun under a load equal to elastic limit, will continue under a less load.

* *Van Nostrand's Magazine*, November, 1877, p. 459.

† *Philosophical Transactions*, Royal Soc. of London, Vol. 210, p. 35.

‡ "Compressive Strength of Steel and Iron," *Transactions*, Am. Soc. C. E., Vol. XVII (1887), pp. 56–57.

"The plotted tests in tension were all so made as to develop the amount of this stretch; and it is demonstrated in the cases of the 100 000-lb. steel, the 67 000-lb. steel, and the hard iron made from scrap, that the total is obtained equally as well, though requiring greater length of time, with a load 1 000 or 500 lb. less than primitive elastic limit as with a load equal to that limit. This becomes clear from the diagrams, when it is understood that the loads just above the limit were each allowed a minute or more in which to develop stretch."

* * * * * * * *

"The amount of stretch at elastic limit as taken from the tension diagrams:

"For the 100 000-lb. steel.......................... 0.3 per cent.
" " 80 000 " " .... 1.4 " "
" " 67 000 " " .... 1.85 " "
For hard iron.................................. 1.3 " "
" soft iron .... 2.3 " " "

The reduction in the load in the extreme case was about 7 per cent.

TABLE 9.—Differences in Recorded Results of Tests Made at Different Laboratories of Specimens of Carbon Steel from the Same Plates.*

(*a*) Yield Point and Drop of Beam.

Laboratory at which tested.	Point observed.	Speed of testing machine.†	Load per Square Inch at Point Observed.					
			⅜-in. plate edge.	½-in. plate edge.	¾-in. plate edge.	1-in. plate edge.	1-in. plate interior.	Average of all tests.
"A"	Yield point	1 in. in 6 min.	38 800	36 880	33 420	......	......	32 400
		1 in. in 20 min.	......	......	......	27 380	25 700	
"A"	Drop of beam	1 in. in 6 min.	40 800	37 320	34 050	......	......	33 670
		1 in. in 20 min.	......	......	......	29 230	26 950	
"B"	Drop of beam	1 in. in 3 min.	Interior 38 000	Interior 33 900	Interior 30 900	......	28 500	32 800
"C"	Drop of beam	1 in. in 15 sec.	46 700	40 800	36 400	......	38 500	40 600

(*b*) Tensile Strength.

Material.	⅜-inch plate.			½-inch plate.			¾-inch plate.			1-inch plate.		
Speed, 1 in. in:	6 min.	3-2 min.	15-10 sec.	6 min.	3 min.	15-10 sec.	6 min.	3 min.	15-10 sec.	6 min.	3-2 min.	15-10 sec.
Tensile strength at edge, in pounds per square inch	......	63 700	......	63 800	......	......	62 200	......	......	......	61 900	......
Tensile strength in interior, in pounds per square inch	......	62 400	66 100	......	62 000	65 300	......	61 100	62 700	59 000	58 300	61 600

* As given by J. A. L. Waddell, M. Am. Soc. C. E., for a permanent set of 0.01 in. in 8 in. Was assumed by Waddell to indicate the yield point, *Transactions*, Am. Soc. C. E., Vol. LXIII (1909), p. 222.

† At yield point and drop of beam.

From tests conducted at the Watertown Arsenal and published in its reports for the period, 1881-1910, results given in Table 10 have been selected

as illustrative of extreme cases of drop in load and as suggestive of its significance. In every one of the tests constituting the first set given in Table 10 the load dropped to either 41 000 or 42 000 lb. per sq. in.; and it is reasonable to infer that the points to which the loads dropped in these ten tests are for them the real criteria of utilizable capacity.

TABLE 10.—COMPARISON OF TENSILE STRENGTH, ELASTIC LIMIT, U. L. P., AND POINT TO WHICH LOAD DROPPED.

(Tests 7737-7745, steel bars for comparison of testing machines in Germany; gauged length, 8 in.; comp. from Watertown Arsenal Report, 1903, pp. 455-467. Tests 7934-7936, steel specimens for comparison of testing machine at works of Vicker's Sons and Maxim (Limited), London, England; gauged length, 8 in.; comp. from Watertown Arsenal Report, 1904, pp. 197-199. Up to the elastic limit the elasticity was as nearly perfect as could be gauged by micrometer readings to 0.0001 in. The load dropped immediately after reaching the U. L. P.)

Test No.	Tensile strength, in pounds per square inch.	Elastic limit, in pounds per square inch.	U. L. P.*		Load Dropped to:		
			In pounds per square inch.	Percentage of tensile strength.	Pounds per square inch.	Percentage of tensile strength.	Percentage of U. L. P.
7737	65 360	46 000	47 000	71.9	41 000	62.7	87.2
7738	64 680	50 000	51 000	78.9	41 000	63.4	80.4
7739	65 200	47 000	47 640	73.1	41 000	62.9	86.1
7740	64 740	49 000	49 000	75.7	42 000	64.9	85.7
7741	65 000	49 000	49 360	75.4	42 000	64.6	85.1
7742	65 080	49 000	50 200	77.1	41 000	63.0	81.7
7743	64 560	47 000	47 600	73.7	42 000	65.1	88.2
7744	64 780	51 000	51 400	79.3	41 000	63.3	79.8
7745	64 840	48 000	48 940	75.5	42 000	64.8	85.8
7746	64 260	47 000	47 560	74.0	41 000	63.8	86.2
7934	48 240	40 000	40 600	84.2	30 000	62.2	73.9
7935	69 080	51 000	51 000	73.8	46 000	66.6	90.2
7936	107 200	†55 000	†58 600	54.7	55 000	51.3	93.9
From Table 5	68 000	....	51 000	75.0	42 000	61.8	82.4

* In each case, the U. L. P. was obtained from deformation given in the reports of the Watertown Arsenal, but the term does not appear therein.

† The elastic limit may have been higher and U. L. P. lower, as no data are given between 55 000 and 58 600 lb. per sq. in.

As a full-sized structural member usually lacks the homogeneity of a specimen, there is little chance, if any, that in a test the load on such a member, after reaching its U. L. P., will fall much below it, except during subsequent failure by bending or local crippling. If it ever occurs, it would be most likely in the case of eye-bars. The report of the Watertown Arsenal for 1883 shows tests of six eye-bars; that for 1886, tests of twelve eye-bars; and, that for 1901, the test of one eye-bar. Some of these eye-bars showed pronounced yield points but the load in no case dropped below the U. L. P. previously indicated. In its report the Special Committee on Steel Columns and Struts states*:

* *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1618.

"In straining a column, there is a point beyond which its structural value is uncertain, and consequently unsafe to rely upon. This point lies somewhere above the region of perfect elasticity and well below the place where manifest yielding occurs. For the study of column tests, the U. L. P. as above defined seems to fulfill these conditions satisfactorily. Careful observations and plotting of the stress-strain curve locate it without chance for controversy. It will be noted that the method is applicable to both tension and compression tests."

As far as was possible the Committee determined the U. L. P.'s of specimens of the shapes and flats composing the columns, and compared them with the tests of the full-sized members. From these tests, and all the foregoing data (in particular those recorded in Table 10), the writer has concluded that tensile tests of specimens of ordinary size, representative of the steel in a given column, conducted with the care and precision characteristic of the engineers of the U. S. Bureau of Standards and the Watertown Arsenal, are very valuable criteria of the strength of the column; and that they are better criteria than tests by compression of individual shapes and plates representative of those of which the column is composed; and he has devised the following:

Rule for Determining Critical Value.—The critical value of each specimen test in tension, of steel of good quality, as a criterion of the U. L. P. of a full-sized tension or short compression member of a structure, shall be considered as either 87% of its own U. L. P., or 60% of its tensile strength, whichever is the smaller.

As to the use of the specimen tests as criteria, the following is suggested:

Rule for Finding the U. L. P. of Built-Up Members from Specimen Tests.—To gauge the U. L. P. of a full-sized tension or short compression member, from tensile specimen tests, divide the area of its cross-section into representative parts; make specimen tests of each part and find the critical value of each specimen from its U. L. P. or tensile strength according to the foregoing rule; multiply the average of the tests for each part by its area, add the results, and divide their sum by the total area. The final result is the weighted average of the critical values of the specimens and indicates the U. L. P. under simple compression or tension.

This latter rule applies to properly designed, properly fabricated, centrally loaded members, made of steel of good quality, as nearly free from flexure as is practicable in tests of full-sized members. In practice, flexure is rarely confined to these limits.

Part 6.—Utilizable Capacity of Compression Members

For convenience, all compression members of structures are herein termed "columns".

Each tension or compression member of a structure has some undesigned eccentricity of its axis with reference to the line of force herein termed "unintentional eccentricity". In addition, there may be intentional eccentricity; but Part 6 deals exclusively with columns to which the load is applied longitudinally and without intentional eccentricity. (For combined compression and flexure, see Part 9.)

In practice even short columns are liable to considerable unintentional eccentricity. The U. L. P. of a column depends partly on the character of

the steel of which it is composed and partly on the unintentional eccentricity. The latter in suitable tests of centrally loaded short columns is so small that the U. L. P. may be considered as due to the character of the steel for resistance to force.

In this paper the term, "characteristic U. L. P.", is used to designate — in the steel composing a column—the property of resisting force, measured by the U. L. P., the column, if short, would have in a suitable compression test. In the absence of such tests the characteristic U. L. P. may be inferred from suitable tensile specimen tests.

A comparison of the observed U. L. P.'s of the short full-sized columns tested for the Special Committee on Steel Columns and Struts, with the U. L. P.'s inferred from tensile tests of specimens, as described in Part 5, is given in Table 11.

For the purposes of analysis, columns are classified as short, medium, and long. In the analysis hereinafter given, the utilizable capacities of short, medium, and long columns are determined by different equations corresponding to different conditions which for different ranges of length are critical. The points of intersection of the loci of the different equations define the limits to the ranges of length to which they apply. The divisions between these classes, and the influence of length on the utilizable capacity of medium columns, vary with each different U. L. P.

For the purpose of determining the utilizable capacities of columns both tests and analyses are desirable. When they harmonize the analysis explains the results of the tests and the tests confirm the results of the analysis. Other things being equal, full-sized tests are better criteria than analyses, but where sound analyses based on reasonable assumptions as to the unintentional eccentricity and the end conditions indicate smaller utilizable capacities than tests, analyses are the better criteria for the reason that the conditions which obtain in tests are generally less severe than those to which columns are liable in practice.

Columns are never absolutely straight. It may happen that the crucial eccentricity between the axis and the line of force is due to an initial curve in the axis. If the largest reasonable initial curve in the axis is assumed, it will lead to smaller computed utilizable capacities for medium and long columns than those computed from alternative reasonable possibilities. For short columns alternative reasonable possibilities are crucial.

In practice nearly all columns or compression members of structures are connected to adjacent members by full riveted joints, butt joints, or pins; and the joints have sufficient stiffness to transmit flexure between the members, except in the case of pin-ended members under intentional flexure of sufficient magnitude to overcome friction on the pins.

For short columns with butt joints, such as are common in practice, if the abutting faces are not machined to the proper angles a part of the load (sufficient to deform the columns to a point where the faces are in suitable contact) will act with great eccentricity, and the shorter the column the greater

TABLE 11.—USEFUL LIMIT POINTS OF SHORT COLUMNS TESTED BY THE U. S. BUREAU OF STANDARDS FOR THE SPECIAL COMMITTEE ON STEEL COLUMNS AND STRUTS. COMPARED WITH MAXIMUM TENSION AND DROP OF BEAM IN SPECIMEN TESTS* AND THE U. L. P.'S DERIVED FOR COLUMNS FROM THE CRITICAL VALUES OF SPECIMEN TESTS BY THE RULE AT THE END OF PART 5.

(All U. L. P.'s and stresses in thousands of pounds per square inch.)

Type.			Specimen Tests.						Full-Sized Tests of Short Columns.					
			Maximum Tension.		Drop of Beam.		Derived from Critical Values.		Average.					
										Percentages.				
No.	Section.	Weight.	No. of tests.	Stress.	No. of tests.	Stress.	No. of tests.	U. L. P.	U. L. P.	Of derived U. L. P.	Of drop of beam.	Of maximum tension.	Minimum U. L. P.	Maximum U. L. P.
1		Light	7	58.9	7	34.0	8	29.3†	28.3	96.6	83.2	48.0	26.0	30.0
		Heavy	7	59.5	7	38.3	2	25.9	25.0	96.5	65.3	42.0	24.0	25.7
		Extra heavy	4	59.6	4	36.4	1	27.1	26.2	96.7	72.0	44.0	26.0	26.5
2	3¼″	Light	5	57.7	5	36.5	8	28.0†	28.1	100.4	77.0	48.7	26.9	29.2
		Heavy	4	59.7	4	36.9	6	26.8†	26.8	100.0	72.6	44.9	26.0	27.5
3	5″	Light	5	55.6	5	39.8	4	28.6	28.2	98.6	70.9	50.7	26.9	29.0
		Heavy	2	59.2	2	37.9	No	data	25.5	.....	67.3	43.1	25.0	26.0
4		Light	5	58.1	5	36.5	18	29.8†	31.6	106.0	86.6	54.4	29.9	33.0
		Heavy	4	59.4	4	37.7	13	22.9†	22.9	100.0	60.8	38.6	22.0	24.0
5		Light	2	61.5	No	data	3	29.2†	33.8	‡	....	55.0	32.0	35.0
		Heavy	3	57.9	"	"	3	29.0†	30.9	106.6	....	53.4	29.3	32.5
		Extra heavy	12	56.4	"	"	12	20.0†	20.0	100.0	....	35.5	19.0	21.1
6		Light	9	57.6	9	34.6	No	data	24.5	.....	70.6	42.5	23.7	26.2
		Heavy	3	60.4	3	38.3	"	"	22.6	.....	59.0	37.4	21.0	23.5
7		Light	5	59.4	5	34.6	3	29.1	30.0	103.1	86.7	50.5	28.5	31.0
8		Light	6	56.3	6	39.2	No	data	31.2	.....	79.6	55.4	30.0	32.5
		Heavy	3	58.7	3	37.6	"	"	27.8	.....	§	‖	20.0	31.0
10		Light	16	58.1	16	37.6	"	"	26.8	.....	71.2	46.1	26.0	27.7
		Heavy	3	61.7	3	37.6	"	"	25.2	.....	67.0	40.8	24.5	26.5

NOTES.—For each type of column, the maximum, minimum, and average U.L.P. of six columns are given; three with $\frac{l}{r} = 50$, and three with $\frac{l}{r} = 85$, except for Type 1 Extra Heavy for which only three were tested $\left(\text{with } \frac{l}{r} = 85\right)$. Omitting these three from consideration, the U.L.P.'s of columns with $\frac{l}{r} = 85$ is smaller than for columns with $\frac{l}{r} = 50$ by less than 1 per cent. Where the maximum or minimum U.L.P. is for a column with $\frac{l}{r} = 85$, it is given in bold-face type.

*Comp. from Final Report of Special Committee on Steel Columns and Struts, *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), pp. 1583 *et seq.*
† Weighted average. The other averages of critical values are simple. ‡ The critical value of one of the three specimens of Type 5 Light was only 22.3 which is abnormal. § The minimum full-sized test of Type 8 Heavy is abnormal, but is included in the average. ‖ Percentage omitted because of abnormality of one of three tests included in average.

will be that part.* Another matter of interest in considering short columns is the fact that even when fairly well designed they are liable in some details to local secondary stresses or reduction in stiffness, which may control their utilizable capacity. In consideration of these facts it is recommended that in practice the utilizable capacity of a short column should be considered as only 90% of the U. L. P. indicated by critical values of specimen tests or by tests of short columns of the same steel and cross-section. Where a large girder or floor-beam, liable to heavy loading, is riveted to a column, the flexure transmitted to the column may be so great that its utilizable capacity to resist direct compression may be reduced to less than 90%; but such cases are special and should receive particular consideration.

Let,

u = the U. L. P. of a tension member or short column, as determined by specimen tests as described in Part 5; or as determined by full-sized tests of tension members or short columns of the same material and cross-section; and

c = the utilizable capacity per unit of cross-sectional area of a column.

For short columns,

$$c = 0.9\,u \qquad (1)$$

The mathematical analyses which follow were made for columns with pivoted or frictionless hinged (but not free) ends, and result in Equation (6) for long columns and Equation (11) for medium columns. Following these analyses, it is shown that it is advisable to apply these equations modified or unmodified, according to conditions, to columns with pin, flat, or riveted ends.

For convenience of analysis and discussion the moments and bending stresses corresponding to the deviation of the line of force from the axis of a structural member before deflection are termed primary moments and stresses, while those corresponding to the deflection (additional deviation) are termed secondary moments and stresses. For further convenience of analysis the curve by which the axis is assumed to deviate initially from the line of force, is assumed to be a sinusoid. Even with this further assumption the analysis is somewhat complicated, and its results are difficult to apply with precision to medium columns; but following the analysis, simple empirical formulas for medium columns are given as alternatives and shown to be nearly equivalent as regards results. (See Equation (11) and Table 12.)

For the benefit of those who desire to follow the general argument without following the mathematical analyses by which Equation (11) for medium columns and Equation (6) for long columns were derived, it may be noted that all medium and long columns are assumed to have an unintentional eccentricity of $\frac{1}{700}$ of their length between pins or points of contrary flexure and that the utilizable capacities of long columns are limited to the loads which would produce deflections of $\frac{1}{300}$ of such length.

* "Imperfect Butt-Joints in Columns, and Stress in Lattice-Bars", *Engineering News*, October 3, 1907, p. 368.

TABLE 12.—CHARACTERISTIC USEFUL LIMIT POINTS, u, OF COLUMNS, BEYOND WHICH PLASTIC DEFORMATION IS SERIOUSLY DETRIMENTAL. LIMITING RATIOS, $\frac{l}{r}$, BETWEEN WHICH COLUMNS ARE "MEDIUM"; BELOW WHICH THEY ARE "SHORT"; AND ABOVE WHICH THEY ARE "LONG". UTILIZABLE CAPACITY, c, OF LONG COLUMNS BEYOND WHICH ELASTIC DEFLECTION IS LIABLE TO BE EXCESSIVE (MORE THAN $\frac{l}{300}$).

For short columns, the utilizable capacity, $c = 0.9\,u$ .... (1)

For medium columns, $c = u - \left(n_1 \frac{l}{r} + n_2 \frac{l^2}{r^2} \right)$ .... (11)

For long columns, $c = \dfrac{207\ 260\ 000}{\left(\frac{l}{r}\right)^2}$ .... (6)

Assuming initial eccentricity $= \frac{l}{700}$; ends pivoted.

u, in pounds per square inch.	MEDIUM COLUMNS.			LONG COLUMNS.		u, in pounds per square inch.	MEDIUM COLUMNS			LONG COLUMNS.	
	$\frac{l}{r}$.	n_1.	n_2.	$\frac{l}{r}$.	c, in pounds per square inch.		$\frac{l}{r}$.	n_1.	n_2.	$\frac{l}{r}$.	c, in pounds per square inch.
19 505	41.6	38.4	0.205	154	8 739	30 493	39.6	50.3	0.680	114	15 948
19 691	41.5	38.6	0.211	153	8 854	30 906	39.5	50.6	0.705	113	16 232
19 880	41.5	38.9	0.217	152	8 971	31 328	39.4	50.9	0.730	112	16 523
20 071	41.5	39.2	0.223	151	9 090	31 760	39.4	51.2	0.756	111	16 822
20 264	41.4	39.4	0.229	150	9 211	32 203	39.3	51.5	0.781	110	17 129
20 465	41.4	39.7	0.235	149	9 336	32 657	39.2	51.7	0.806	109	17 445
20 665	41.3	40.0	0.242	148	9 462	33 121	39.2	52.0	0.837	108	17 769
20 870	41.3	40.3	0.249	147	9 591	33 599	39.1	52.2	0.868	107	18 103
21 079	41.3	40.5	0.256	146	9 723	34 088	39.0	52.5	0.899	106	18 446
21 291	41.2	40.8	0.263	145	9 857	34 590	38.9	52.7	0.930	105	18 799
21 509	41.2	41.1	0.270	144	9 995	35 105	38.8	53.0	0.968	104	19 162
21 729	41.1	41.4	0.278	143	10 135	35 634	38.8	53.2	1.006	103	19 536
21 956	41.1	41.6	0.287	142	10 279	36 177	38.7	53.4	1.044	102	19 921
22 184	41.1	41.9	0.295	141	10 425	36 735	38.6	53.6	1.082	101	20 318
22 417	41.0	42.2	0.304	140	10 574	37 307	38.5	53.8	1.120	100	20 726
22 655	41.0	42.5	0.312	139	10 727	37 895	38.4	53.9	1.169	99	21 147
22 898	40.9	42.8	0.322	138	10 883	38 499	38.3	53.9	1.218	98	21 580
23 146	40.9	33.1	0.331	137	11 043	39 120	38.3	54.0	1.267	97	22 028
23 398	40.8	43.5	0.341	136	11 206	39 762	38.2	54.0	1.316	96	22 490
23 653	40.8	43.8	0.350	135	11 372	40 417	38.1	54.1	1.364	95	22 964
23 917	40.7	44.1	0.360	134	11 543	41 095	38.0	54.2	1.421	94	23 456
24 216	40.7	44.4	0.372	133	11 717	41 791	37.9	54.3	1.478	93	23 963
24 456	40.6	44.7	0.384	132	11 895	42 511	37.8	54.3	1.540	92	24 488
24 782	40.6	45.0	0.396	131	12 076	43 250	37.7	54.2	1.604	91	25 029
25 019	40.5	45.2	0.408	130	12 264	44 011	37.6	54.2	1.673	90	25 588
25 309	40.5	45.5	0.420	129	12 455	44 798	37.5	54.1	1.752	89	26 167
25 604	40.4	45.8	0.434	128	12 650	45 604	37.3	54.0	1.831	88	26 763
25 906	40.4	46.1	0.448	127	12 850	46 442	37.2	53.9	1.910	87	27 383
26 214	40.3	46.5	0.462	126	13 055	47 305	37.1	53.8	1.989	86	28 024
26 530	40.3	46.8	0.476	125	13 265	48 194	37.0	53.7	2.068	85	28 687
26 852	40.2	47.1	0.490	124	13 480	49 112	36.9	53.3	2.173	84	29 373
27 181	40.1	47.4	0.507	123	13 700	50 063	36.8	52.8	2.279	83	30 086
27 516	40.1	47.7	0.524	122	13 925	51 045	36.7	52.4	2.384	82	30 824
27 859	40.0	48.0	0.541	121	14 156	52 062	36.5	51.9	2.490	81	31 591
28 210	40.0	48.4	0.558	120	14 393	53 110	36.4	51.4	2.596	80	32 384
28 569	39.9	48.7	0.575	119	14 636	54 197	36.3	50.6	2.736	79	33 209
28 936	39.8	49.0	0.595	118	14 885	55 325	36.1	49.8	2.877	78	34 067
29 313	39.8	49.4	0.615	117	15 141	56 489	36.0	49.0	3.017	77	34 956
29 697	39.7	49.7	0.635	116	15 403	57 700	35.9	48.2	3.158	76	35 883
30 090	39.6	50.1	0.655	115	15 672	58 955	35.7	47.4	3.298	75	36 847

Let,

l = length.

I = moment of inertia.

r = radius of gyration.

p = load per unit of cross-sectional area of column.

e = eccentricity of the line of force relative to the axis of the column at the center before deflection.

$\varDelta$ = deflection at the center of the column.

f = the intensity of the combined stress from direct compression and bending at the center of the column in the extreme fiber on the concave side.

$$E = \text{Modulus of elasticity} = \frac{\text{Direct stress intensity}}{\text{Proportionate elongation or shortening}}.$$

For steel, considering the pound as the unit of stress and the inch as the unit of length, $E = 30\,000\,000$.

Let,

v = the distance from the axis of the column to the extreme fiber on the concave side.

$$q = \frac{\pi^2 E r^2}{l^2}$$

which is Euler's formula.

The curve of the primary bending moment will be sinusoid. It is a peculiarity of a sinusoidal moment diagram that the deflection corresponding to it will be a sinusoid; hence, the secondary bending moment will likewise be a sinusoid.

From the theory of flexure, when the moment diagram is a sinusoid,

$$\varDelta = \frac{\text{Bending moment} \times l^2}{\pi^2 E I} = \frac{p\,e\,l^2}{\pi^2 E r^2} + \frac{p\,\varDelta\,l^2}{\pi^2 E r^2} = \frac{p\,e}{q-p} \ldots\ldots (2)$$

$$f = p + \frac{p\,e\,v}{r^2} + \frac{p\,e\,v}{r^2}\left(\frac{p}{q-p}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Let,

$\frac{l}{x}$ = the unintentional eccentricity.

The utilizable capacity of long columns is limited to the load which can be supported without excessive deflection.

Let,

$\frac{l}{y}$ = the greatest deflection which is not excessive.

Substituting these values in Equation (2),

$$\frac{l}{y} = \frac{p\,\frac{l}{x}}{q-p} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

from which,

$$p = q\,\frac{x}{y+x} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Assuming that $\frac{l}{x} = \frac{l}{700}$, which seems ample for unintentional eccentricity, and that, for the condition ($p = c$), $\frac{l}{y} = \frac{l}{300}$ (which although great, is not, for a column loaded to its utilizable capacity, an excessive deflection) then, in pounds per square inch,

$$c = 0.7\,q = \frac{207\,260\,000}{\left(\frac{l}{r}\right)^2} \quad \text{.........................(6)}$$

Equation (6) gives the utilizable capacities in pounds per square inch of long steel columns of good workmanship with pivoted or frictionless hinged ends. It is independent of u or of any other critical stress and gives values for c which vary solely as the inverse of $\left(\frac{l}{r}\right)^2$. If a column qualifies as a "long" one, its utilizable capacity will in nowise depend on what U. L. P. a short column of similar steel and cross-section would possess.

Medium columns are classified according to the U. L. P.'s that short columns of similar steel and cross-section would possess. In this paper the U. L. P. that determines the series is termed the "characteristic U. L. P.", or "characteristic u".

For medium columns the utilizable capacity, c, may be considered to be the load that will produce a maximum stress intensity, f, equal to the characteristic u. The dividing line between medium and long columns is the $\frac{l}{r}$ at which the load that will cause a deflection, $\frac{l}{300}$, will also cause a stress, u, in the critical extreme fiber. At this line, from Equations (3) and (6), for $e = \frac{l}{700}$ and $\frac{v}{r} = 1.68$,

$$u = c + \frac{1.68\,c\,l}{700\,r} + \frac{1.68\,c\,l}{700\,r} \times \frac{7}{3} = c + \frac{0.008\,c\,l}{r} \quad \text{...........(7)}$$

Equation (7) determines the characteristic u of the series; it is useful in compiling tables.

For ratios of v to r other than 1.68, Equation (7) is approximate; but the error is not great. In most cases the ratio is less than 1.68 and, therefore, the dividing line between short and medium columns occurs at a slightly greater value of $\frac{l}{r}$ than that for a ratio of 1.68.

From Equation (1) the dividing line between short and medium columns is the $\frac{l}{r}$ at which a load of 0.9 u will produce a stress, f, equal to u.

For the conditions, $e = \frac{l}{700}$, $\frac{v}{r} = 1.68$, and $p = 0.9\,u$,

$$\frac{p\,e\,v}{r^2} = \frac{0.9\,u\,e\,v}{r^2} = \frac{0.9\,u\,\frac{l}{700}\,1.68\,r}{r^2} = 0.00216\,u\,\frac{l}{r} \quad \text{.........(8)}$$

$$u = 0.9\,u + 0.00216\,u\,\frac{l}{r} + 0.00216\,u\,\frac{l}{r}\left(\frac{0.9\,u}{q - 0.9\,u}\right) \ldots\ldots\ldots (9)$$

and, solving,

$$\frac{l}{r} = \frac{46.296}{1 + \dfrac{0.9\,u}{q - 0.9\,u}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (10)$$

From Equation (10), the $\frac{l}{r}$, that marks the dividing line between short and medium columns of any series can be found; it has to be solved by trial, which is tedious but does not impair its usefulness in compiling tables.

The limits expressed in terms of the $\frac{l}{r}$ within which columns are "medium" are given in Table 12. In compiling these limits, the dividing lines between medium and long columns were chosen, and the characteristic u was determined for each series from Equation (7). The limiting $\frac{l}{r}$ for each series as between medium and short was then determined from Equation (10).

At the limits of the "medium" columns in each series, the utilizable capacity can be found from Equations (1) and (6). For intermediate values of $\frac{l}{r}$ an exact equation can be derived from Equation (3), but it would be far too complicated for ordinary use. However, within the limits of medium columns, the loci of the exact equation can be approximated so closely by an equation of the second degree that the differences between the accurate and approximate loci are of no practical importance. Such an equation has the form:

$$c = u - \left(n_1\,\frac{l}{r} + n_2\,\frac{l^2}{r^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

in which, n_1 and n_2 are numerical factors the values of which are given in Table 12 for each series. They were determined by reduction after placing Equation (11) equal to Equations (1) and (6) at the respective values of $\frac{l}{r}$ for the dividing points between medium and long columns of each series.

Table 12 gives for medium columns the limiting values of $\frac{l}{r}$ and values of n_1 and n_2 for eighty successive characteristic U. L. P.'s from u = 19 505 lb. per sq. in. to u = 58 955 lb. per sq. in. It also gives the utilizable capacity, c, of long columns from $\frac{l}{r} = 75$ to $\frac{l}{r} = 154$, inclusive.

For the assumed conditions that unintentional eccentricity, e, $= \frac{l}{700}$ and $\frac{v}{r} = 1.68$, the writer has made a considerable number of computations to develop the maximum error involved in using Equation (11) for determining

the utilizable capacity, c, in place of determinations by trial from Equation (3). The maximum error discovered was an excess in capacity indicated by Equation (11) of 2% in the case of a column with a u of 20 071 lb. per sq. in. and an $\frac{l}{r}$ of 120. The greatest errors occur in the series having the smallest values of u. All the errors are probably well within 3 per cent.

To summarize: From Table 12 any column with pivoted or frictionless hinged ends can be readily classified as short, medium, or long, when its characteristic U. L. P. is known; if short, its utilizable capacity can be found from Equation (1); if medium, from Equation (11) with suitable values of n_1 and n_2 taken from Table 12; and if long, from Equation (6), or directly from Table 12 if its $\frac{l}{r}$ is any whole number from 75 to 154, inclusive.

The series of tests of columns with flat ends made by the U. S. Bureau of Standards for the Special Committee on Steel Columns and Struts, and the numerous series of tests of centrally loaded pin, flat, spherical, and fixed-ended columns made since 1880 by U. S. Government Engineers at the Watertown Arsenal, developed in all but one series a much smaller reduction in utilizable capacity with increase in length than that indicated by Equations (6) and (11) (as shown in Tables 11 and 13).

In tests of pin-ended columns, the friction between the pins and their bearings restrains the ends and causes the column to act as if it were fixed or flat-ended until the tendency (induced by the primary and secondary bending moments) of the ends to turn on the pins is sufficient to overcome the friction. In all but one case the records of the tests of pin-ended columns cited in Table 13, for which $\frac{l}{r}$ was greater than 96, show that the ends were restrained up to the moment of failure and that the columns then deflected suddenly, thus indicating that the ends suddenly turned on the pins. In the one case, in which the record makes no mention of sudden deflection, the unintentional eccentricity from an initial curve in the axis $\left(\frac{\text{length}}{550}\right)$ was greater than the assumption of $\frac{\text{length}}{700}$, and the deflection at the U. L. P. $\left(\frac{\text{length}}{241}\right)$ exceeded the permissible deflection of $\frac{\text{length}}{300}$, on which Equation (6) is based. The ends in this one case probably turned on the pins before the U. L. P. was reached, and the load at the "permissible deflection" was probably less than the 9 200 lb. per sq. in. indicated by Equation (6).

The additional value of c, if any, which a medium or long column in service possesses over and above the value indicated by Equations (6) or (11), by reason of its having pin, flat, or riveted ends, depends in each case on the conditions.

In laboratory tests the pin and flat ends of columns are restrained by a fairly rigid testing machine, whereas in practice the ends are restrained simply

by the stiffness of adjoined more or less flexible members of a deflected structure, the latter, probably, being much less effective. In pin-ended members the ability of adjacent members to stiffen a member is contingent on friction on the pins, which will be overcome if the bending moment is large enough. For long columns and, ordinarily, for medium columns, it is advisable not to place any reliance on friction but to classify as "frictionless pin or hinged-ended" all columns with pin ends and all continuous compression chords of trusses having pin connections to the web members. For long and medium columns with riveted end connections to adjacent members some allowance may be made for the restraining influence of the end connections by letting the l of the equations and of Table 12 represent from three-quarters to two-thirds of the length, according to the conditions, instead of the whole length as in the case of columns with pivoted or frictionless hinged ends. In other words for such columns l represents the distance between points of contrary flexure assumed by judgment. Under ideal conditions, l for columns with fixed ends equals one-half the length, and for columns with one free and one fixed end twice the length; but ideal conditions are not achieved even in the laboratory.

To illustrate the use of Table 12 and Equation (11), a nickel steel column with a characteristic u of 47 700 lb. per sq. in. and an $\frac{l}{r}$ of 100, well connected to adjacent fairly stiff members by riveted joints, might be assumed to have an $\frac{l}{r}$ between points of contrary flexure of 66.7 and, from Table 12, would be a medium column with an n_1 of 53.7 and an n_2 of 2.033 and, from Equation (11), would have a utilizable capacity, c, of 35 100 lb. per sq. in. The test of such a column with flat ends (cited in a footnote to Table 6) showed under laboratory conditions very little variation from perfect elasticity up to 47 000 lb. per sq. in. and a failure load of 48 910 lb. per sq. in.

The equations and Table 12 given in this Part 6 are based on the assumption of no intentional eccentricity between the line of thrust and the axis of the column, whether produced by eccentric application of the longitudinal load or by a combination of longitudinal and transverse loads. Where there is intentional eccentricity, allowance should be made as described in Part 9. The weight of horizontal and inclined columns causes intentional eccentricity.

It is essential to the applicability of Equations (1), (6), and (11) that the details shall be able to develop the capacity indicated by the equations. For example, tests of forty-two columns by the U. S. Bureau of Standards, for the American Railway Engineering Association's Committee on Steel and Iron Structures, included comparative tests of columns with lacing and columns with batten-plates, and the report of that Committee states:*

"In all cases the columns with batten-plates failed at an ultimate strength of from 6 000 to 9 000 lb. per sq. in. less than the corresponding column with lacing, and, in one case, the heavy section with 120 $\frac{l}{r}$ as much as 15 000 lb. per sq. in. less."

* *Proceedings,* Am. Ry. Eng. Assoc., Vol. 19, 1919, p. 960.

TABLE 13.—COMPARISON OF USEFUL LIMIT POINTS AND UTILIZABLE CAPACITIES, c, IN FULL-SIZED COLUMN TESTS.

(Sets 1, 2, 3, and 4 were compiled from Watertown Arsenal Reports for 1908, 1909, 1910, and 1883; Set 5 from *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-1920).

All U. L. P.'s are in 1 000 lb. per sq. in. All deflections (except one) were less than $\frac{\text{Length}}{300}$. In most cases the average is the average U. L. P. of three columns.

For all pin-ended columns (except one) with $\frac{l}{r}$ greater than 96, the reports record sudden lateral springing at maximum load.)

(a) LAP-WELDED BESSEMER STEEL TUBING, 5 INCHES IN NOMINAL DIAMETER; 5.02 TO 5.36 SQUARE INCHES IN SECTIONAL AREA; AVERAGE CHARACTERISTIC U. L. P. PER SQUARE INCH = u = 31 000 POUNDS PER SQUARE INCH.

SPHERICAL ENDS.				PIN ENDS.				FLAT ENDS.				FIXED ENDS.				FROM TABLE 12.	
$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	Minimum.	Maximum.	Average.	$\frac{l}{r}$.	c.
47	31.0	32.0	31.7	47	31.0	33.0	32.3	50	28.0	31.0	29.5	44	29.0	32.0	30.3	47	27.9
72	32.0	34.0	33.3	72	29.0	31.0	30.3	75	27.5	32.0	30.5	69	29.0	35.0	31.7	72	23.7
97	28.0	31.0	29.3	97	22.5	31.0	27.5	100	27.0	30.0	28.7	93	28.0	32.0	30.0	97	19.7
150	23.0	27.0	25.3	150	25.0	25.0	25.0	150	23.0	26.0	25.0	146	24.0	30.0	26.3	150	9.2
175	24.0	25.0	24.7	175	19.0	23.7	20.6	175	23.0	27.0	25.3	...	....	....	....	175	6.8

TABLE 13.—(*Continued.*)

(*b*) I-Beams Built of One 10 Inch by ⅜-Inch Plate and Four 4 Inch by 3 Inch by ⅜-Inch Angles.

So-called 60 000-Lb. Steel: $u = 30\,000$ Lb. per Sq In.

$\frac{l}{r}$.	Pin Ends. Minimum.	Pin Ends. Maximum.	Pin Ends. Average.	Flat Ends. Minimum.	Flat Ends. Maximum.	Flat Ends. Average.	c from Table 12.
50	30.0	30.0	30.0	30.0	31.0	30.3	25.9
75	28.0	31.0	29.3	32.0	32.0	32.0	22.6
100	30.0	31.0	30.7	30.0	31.0	30.7	18.4
150	27.0	28.0	27.7	27.0	29.0	28.3	9.2
175	13.0	26.0	20.7	23.0	26.0	24.7	6.8

Nickel Steel: $u = 47\,700$ Lb. per Sq. In.

$\frac{l}{r}$.	Minimum.	Maximum.	Average.	c from Table 12.
25 Flat ends	47.0	48.0	47.7	42.9
100 Flat ends	45.0	47.0	46.0	20.7
117 Pin ends	40.0	40.0	40.0	15.1
187 Pin ends	24.0	27.0	25.7	5.9
214 Pin ends	19.0	19.0	19.0	4.5

So-called 70 000-Lb. Steel: $u = 33\,000$ Lb. per Sq. In.

$\frac{l}{r}$.	Flat Ends. Minimum.	Flat Ends. Maximum.	Flat Ends. Average.	c from Table 12.
25	33.0	33.0	33.0	29.7
100	32.0	32.0	32.0	19.4
...	....	....	....	....
...	....	....	....	....
...	....	....	....	....

(*c*) Steel 6 by 6-In., Rolled H-Sections with Nominal Sectional Area of 7 Sq. In.: $u = 26\,700$ Lb. per Sq. In.

$\frac{l}{r}$.	Pin Ends. Minimum.	Pin Ends. Maximum.	Pin Ends. Average.	Flat Ends. Minimum.	Flat Ends. Maximum.	Flat Ends. Average.	c from Table 12.
25	26.0	26.0	26.0	27.0	28.0	27.7	24.0
50	26.0	27.0	26.3	26.0	27.0	26.7	23.1
75	27.0	27.0	27.0	26.0	27.0	26.7	20.5
100	23.0	26.0	24.7	24.0	25.0	24.7	17.2
150	10.0*	20.0	15.7	23.0	23.0	23.0	9.2

(*d*) Wrought-Iron Built I-Beams between Channels: $u = 29\,500$ Lb. per Sq. In.

$\frac{l}{r}$.	Pin Ends. Minimum.	Pin Ends. Maximum.	Pin Ends. Average.	c from Table 12.
68	29.5	29.5	29.5	23.2
102	21.0	23.5	22.3	17.9
136	16.0	19.0	17.5	11.2

The ratios, $\frac{l}{r}$, are $^{30}/_{28}$ of the actual ratios for comparison with Table 12 for steel.

(*e*) Two Steel Channels and One I-Beam Type 4 Light of Table 11: $u = 31\,600$ Lb. per Sq. In.

$\frac{l}{r}$.	Flat Ends. Minimum.	Flat Ends. Maximum.	Flat Ends. Average.	c from Table 12.
50	29.9	33.0	32.0	27.2
85	30.4	31.5	31.1	21.9
120	28.2	31.6	30.0	14.4
155	21.0	23.0	22.3	8.6
....	....	....	....	....

* Initial curve in axis, $\frac{\text{length}}{550}$; deflection, $\frac{\text{length}}{241}$; when load was reduced to 1 000 lb. per sq. in., it reduced the deflection to 0.03 in.; on reloading, column failed by deflection; maximum load, 10 100 lb. per sq. in.

In some cases the details of a column include lattice-bars for one or two open sides. Even under ideal conditions lattice-bars have to possess considerable potential resistance to shear in order to give the column about as much stiffness against deflection from shear as from bending moment. In actual conditions the lattice-bars actually have to resist shear. Lattice-bars also have to take some of the direct compression and should provide some resistance to possible accidental abnormal transverse loads.

Let,

S = the shear when the column is loaded to its utilizable capacity; and
A = the cross-sectional area of column.

The assumption made for short columns that the column is subject to equal eccentricities in the application of the load on opposite sides of the axis at opposite ends sufficient to produce a bending stress of 0.1 u, and the assumption made for short, medium, and long columns that $\frac{v}{r} = 1.68$, together lead to the equation:

$$S = \frac{u\,A}{8.4\,\frac{l}{r}} \qquad (12)$$

The assumption for medium columns that the stress from bending equals $u - c$, and the assumption for both medium and long columns that the column is initially bent in a sinusoidal bow with unintentional eccentricity of $\frac{l}{700}$, together lead to the equation for medium columns:

$$S = \frac{\pi\,(u - c)\,A}{1.68\,\frac{l}{r}} \qquad (13)$$

The assumption for long columns loaded to their utilizable capacity that the deflection is limited to $\frac{l}{300}$, and the assumption for long columns previously cited, together lead to the equation for long columns:

$$S = \frac{\pi\,c\,A}{210} \qquad (14)$$

The following general rule is suggested for proportioning lattice-bars of columns without intentional bending moment: Proportion the lattice-bars of each open side for a stress of $\frac{0.5\,S}{F}$, in which, F is the factor of safety given by the working load used for the column, and S is obtained from the empirical formula:

$$S = 0.02\,u\,A \qquad (15)$$

Equation (15) gives a value of S varying from 2.1 to 4.5 times as great as either of Equations (13) or (14); and from 0 $\left(\text{when } \frac{l}{r} = 6\right)$ to 6.4 $\left(\text{when } \frac{l}{r} = 38.4\right)$ times as great as Equation (12); hence, in all cases except extremely

short columns, this rule allows considerable margin, for considerations other than the shear indicated by the assumptions on which Equations (12), (13), and (14) are based.

In 1891, the writer suggested the following:*

"The lattice shall be so spaced that each channel between lattice connections shall be stronger than the column considered as a whole, and their size shall be obtained by treating the column as a lattice girder loaded at the middle with a load equal to 3% of the total compression on the column."

He now modifies this suggestion in conformity with the analysis given to the following:

"The lattice shall be so spaced that each channel between lattice connections shall be stronger than the column considered as a whole; and its size shall be obtained by treating the column as a latticed girder loaded at the middle to the amount of 4% of the total compression that would be allowed on the column if short."

PART 7.—UTILIZABLE CAPACITY OF TENSION MEMBERS AND OF MEMBERS SUBJECTED TO ALTERNATE STRESSES; AND THE RESULTS OF LOADING TENSION AND COMPRESSION MEMBERS BEYOND THEIR USEFUL LIMIT POINTS

Let,

t = the utilizable capacity per unit of net cross-sectional area.

Tension members like columns, are liable to unintentional eccentricities between their lines of force and their axes, and their connections are of such stiffness that flexure can be transferred to them from the members to which they are connected. Unlike columns, however, they are not liable to eccentricity from imperfect butt joints, and their flexure, instead of being increased by the secondary moments, is decreased. Another important consideration is the fact that ruinous deformation in most cases probably will not cause failure in a short period, with its resultant danger. Logically, this last consideration affects the factor of safety rather than the utilizable capacity, but to avoid complication, the utilizable capacity is considered, instead, as slightly increased and no recommendation made for any reduction in the factor of safety. After weighing all these facts, the writer considers that except, as noted, $t = u$, u being the useful limit point of a full-sized tension or short compression member as described in Part 5.

The exception is the case of a tension member to which a large girder or floor-beam, liable to heavy loading, is riveted; which, similarly as for columns, is a matter in each case for particular consideration.

There are no published results of tests of full-sized tension members other than eye-bars from which U. L. P.'s can be obtained and no published comparisons of the weighted averages of the U. L. P.'s of specimen tests with the U. L. P.'s of tension members. Such tests are desirable. In their absence it is reasonable to infer that the U. L. P.'s of tension members (per unit of net area), made of single symmetrical shapes or built of shapes and plates, are as great as those of short columns of the same cross-section (per unit of gross area), as inferred from specimen tests.

* *Lehigh Quarterly*, June, 1891, p. 159.

There are no published comparisons between the U. L. P.'s of specimen tests and those of eye-bar tests. For specimen tests to be directly critical as regards U. L. P.'s it would seem that the bars from which specimens are cut should be first annealed, as for eye-bars. In inferring U. L. P.'s of eye-bars from the U. L. P.'s of unannealed specimens allowance should be made for the effect of annealing.

For alternate tension and compression, consider a test piece with a cross-sectional area of 1 sq. in.

Let,

R = the maximum elastic range of stress which can be developed therein.

u = the value of R when the range is from zero to a maximum. For this range, u is the critical value of the specimen.

u' = the greater stress plus x times the lesser stress.......(16)

in which, x is a factor, constant for similar test pieces, but differing for the many grades and kinds of steel.

Let,

y = one-half the stress by which tension exceeds compression or compression exceeds tension, so that,

$$\text{Greater stress} = \frac{R}{2} + y;$$

and

$$\text{Lesser stress} = \frac{R}{2} - y$$

Then,

$$\frac{R}{2} + y + x\left(\frac{R}{2} - y\right) = u'$$

and solving,

$$R = \frac{2}{1 + x}(u' + xy - y) \quad \text{.................. (17)}$$

$$x = \frac{2u' - 2y - R}{R - 2y} \quad \text{.................. (18)}$$

By careful scrutiny and trial, approximate values of u' and x were inferred from Table 1 for Lots 2 and 3, and the elastic ranges were computed by use of Equation (17) and then compared with Table 1 to test the assumption that x "is constant for similar test pieces," as shown in Table 14.

In considering Table 14 it should be remembered that, in the absence of published figures, the writer obtained the developed elastic limits cited in Table 1 by scaling them from Bairstow's diagrams.

In the latter part of Part 5 it is specified that the critical value of each specimen shall be limited to 60% of its tensile strength and to 87% of its U. L. P. If these limitations govern, the value of u for the specimens of Bessemer steel will be limited to 60 000 lb. per sq. in. and x will not be greater than 62%; and for all the specimens cited in Tables 1 and 2, tested with 800 or less "cycles per minute", having a tensile strength of less than 70 000 lb., the value of x will be well within 50 per cent.

There is a clause in some specifications to the effect that members subject to alternate tension and compression shall be proportioned for both tension and compression separately considered, but that each kind of stress shall be considered as increased by 50% of the lesser stress. If the stresses are those obtaining when members are stressed to their utilizable capacity and the metal is 70 000 lb., or less than 70 000 lb., steel, this specification seems to be ample. It is probably ample for all steel, but for steel of more than 70 000 lb. and up to 100 000 lb. maximum tensile strength the writer considers it best to specify percentages of more than 50 and up to 65 per cent.

TABLE 14.—Comparison of Computed with Actual Ranges of Stress to Show in Cases of Alternate Stresses that x in Equation (17) is Constant for Steel of the Same Kind and Grade; y = One-half the Stress by Which Tension Exceeds Compression or *Vice Versa;* and u = Range from Zero to Maximum Stress.

Unannealed Axle Steel (All Stresses are in Pounds per Square Inch; Tensile Strength, 85 600; u = 48 000; x = 64.4 Per Cent.)			Unannealed Bessemer Steel (All Stresses are in Pounds per Square Inch; Tensile Strength, 99 900; u = 66 500; x = 79.2 Per Cent.)		
y.	Ranges:		y.	Ranges:	
	Computed.	From Table 1.		Computed.	From Table 1.
0	58 400	......	0	74 200	......
1 000	58 000	58 000	1 250	73 900	73 900
12 000	53 200	54 000	23 500	68 800	69 000
24 000	48 000	......	31 500	66 900	67 000
27 500	46 500	47 000	33 250	66 500	......

To the writer's knowledge all endurance tests of steel whether by repeated direct tension, by repeated direct compression, or by alternate direct tension and direct compression, are of material that has not been treated by cold-rolling, by cold-drawing, or by heating and quenching. As far as this paper is concerned, the important problem of the utilizable capacity of treated steel eye-bars is not considered.

A superficial consideration of the reports of the Special Committee on Steel Columns and Struts would give the impression that the U. L. P. adopted is the *Ultima Thule* of the structural value of steel. The writer has used it as the limit to the application of mathematical analysis based on the assumption of elasticity. This "point" does indicate the "limit" within which steel is predominantly elastic, and it is very "useful"; the term, "Useful Limit Point", should be thus interpreted, but it does not necessarily indicate a limit beyond which steel ceases to have structural value.

Tests 4135* of a 5 by 1-in. eye-bar, can be considered as typical of the nineteen eye-bars referred to in Part 5 as tested at the Watertown Arsenal. The results of the test of this bar, in a gauged length of 260 in., near its U. L. P. are, as shown in Table 15.

* Watertown Arsenal Report, 1886, p. 1574.

As the gauge was set at zero for 1 000 lb., 0.0091 in. (the elastic elongation for 1 000 lb.) has been added to each elongation taken from the report.

A load about 5% greater than the U. L. P. caused a plastic elongation (which would result in permanent set) of about one-third the elastic elongation; and a load 10% greater caused a plastic elongation of more than 500% of the elastic. Under this latter load, the plastic elongation in 25 ft. 8 in. (the length, center to center, of pins) was 2.3 in. Such a phenomenon in a diagonal bar of a bridge panel 18 ft. high by 18 ft. long would permanently distort the panel 4.6 in.; that is, the vertical drop in the lower pin would be 4.6 in. greater than in the upper one. Such distortion would be ruinous.

TABLE 15.—ELASTIC AND PLASTIC DEFORMATION OF A TYPICAL EYE-BAR.

Pounds per square inch.	Elongation.			
	Total, in inches.	Elastic, in inches.	Plastic.	
			In inches.	In percentage of elastic.
33 920	0.3262	0.3185	0.0077	2.4
35 000	0.3391	0.3283	0.0108	3.3
35 500 U. L. P.	......	......	......	......
36 000	0.3580	0.3374	0.0206	6.1
37 000	0.4231	0.3465	0.0766	22.1
37 200 Approximate	......	......	......	33.3
38 000	0.8591	0.3557	0.5034	145.2
39 000	2.1491	0.3648	1.7843	501.6
69 330 Maximum load	......	......	......	......

The eye-bar had a sharp and pronounced yield with a small increase in load just above the U. L. P. Other members the cross-sections of which consist of one piece are liable to sharp and pronounced yields. For instance an H-section, pin-ended column* had a U. L. P. of 26 000 lb., a linear compression of 0.0306 in. in 20 in. at 31 000 lb. per sq. in. and of 0.23 in. in 20 in. at 32 000 lb. per sq. in. Built members are usually composed of pieces having different U. L. P.'s and yield points, and are subjected to severe local strains in fabrication. They rarely if ever have a pronounced yield that is sharply marked until they are near the point of failure.

To show the behavior of columns when loaded beyond their U. L. P.'s Tables 16 and 17 have been compiled. Table 18 gives data referred to in Table 16. The type, section, and other data concerning these columns are given in Tables 3 and 18. (The columns listed in Table 18 are the largest ever tested, are of many varieties of steel, and the data regarding them are very interesting and valuable as criteria.) The forked ends of the columns indicated as Items 5 and 6 in Table 16 were beyond question critical as regards the maximum loads sustained. It appears from these tables that under laboratory conditions short columns with suitable details, to become permanently shortened by an increment of one-third their elastic shortening under the

* Watertown Arsenal Report, 1909, p. 755.

same load, would on an average require loads about 10% greater than at their U. L. P.'s.

The question arises whether for short built columns and built tension members a total plastic deformation not to exceed one-third of the total elastic deformation would not be a better limit to adopt for critical deformation than the U. L. P. The writer's opinion is decidedly in favor of the U. L. P. Actual conditions are not those of the laboratory; plastic deformation equal to one-third of the elastic deformation would produce excessive permanent distortion of structures and would vitiate the results of computations based on the elasticity of steel.

TABLE 16.—SMALL BUILT COLUMNS, LOADED BEYOND THEIR USEFUL LIMIT POINTS.

R = ratio of plastic to elastic deformation at the U.L.P.
L = load which caused plastic deformation equal to one-third the elastic deformation. After removal of load, plastic deformation becomes permanent set.

References.*	Page.	Description.	Ends.	$\frac{l}{r}$.	U.L.P., in thousands of pounds per square inch.	R approximate percentage.	L. In thousands of pounds per square inch.	L. Percentage of U.L.P.	Maximum. In thousands of pounds per square inch.	Maximum. Percentage of U.L.P.
(1)	802	Average of three carbon..........	Pin	25	30.3	5	34.1	113	37.8	125
(1)	778	" " " "	Flat	25	30.0	15	32.3	108	36.3	121
(1)	781	" " " "	"	50	30.3	3	33.0	109	34.6	114
(2)	154	Average of three 70 000-lb. carbon.	"	25	33.0	5	36.2	110	40.6	123
(2)	145	Average of three nickel..........	"	25	47.7	17	51.3	108	54.5	114
(3)	2452	Type 2 Light (Table 11)..........	"	50	27.0	15	30.0	111	32.6	121
(3)	2452	" " " " "	"	50	29.0	12	32.0	110	34.9	120
(3)	2452	" " " " "	"	50	27.5	7	31.0	113	33.1	120
(3)	1628	Type 4 A Heavy (Table 11)..........	"	50	23.0	19	25.5	111	30.3	132
(3)	1628	" " " " "	"	50	22.5	14	26.5	118	29.0	129
(3)	1628	" " " " "	"	50	22.0	15	25.5	116	28.0	127
(3)	1619	Average of 54 Columns of 18 Types	"	50	27.3	......	......	......	32.7	120
(3)	1619	" " " " " " "	"	85	27.2	......	......	......	30.3	111
(3)	1619	" " " " " " "	"	120	25.8	......	......	......	27.7	107
(1)	808	Average of three carbon..........	Pin	75	29.3	......	......	......	32.4	111
(1)	787	" " " "	Flat	100	30.7	......	......	......	32.4	106
(1)	176	Average of three nickel..........	"	100	46.0	......	......	......	47.6	103

* (1) Watertown Arsenal Report, 1909; (2) Watertown Arsenal Report, 1910; (3) *Transactions*, Am. Soc. C. E., Vol. LXXXIII (1919-20), p. 1583 *et seq.*

PART 8.—BEAMS AND PLATE GIRDERS UNDER TRANSVERSE LOADS

Simple tension or compression in elastic material has its maximum shearing component in any plane making an angle of 45° with the direction of the force. From the theory of mechanics the intensity of this maximum shearing component is one-half that of the simple tension or compression. The microscope has shown that when the elastic limit is reached in malleable metals under simple tension or compression the yield takes place by the sliding within grains of their crystalline layers one over another; which indicates that the material is really yielding under the shearing component of the simple

tension or compression. When, at the elastic limit, iron or steel yields, at an indicated simple tension or compression of, say, 30 000 lb. per sq. in., it actually yields under a shearing component of 15 000 lb. per sq. in. It is a fair inference that under compound stress the elastic limit in iron or steel is reached when the shearing intensity reaches one-half the intensity of simple tension at the elastic limit; and the weight of experimental evidence seems to support this inference. The predominant practice in dealing with beams and plate girders is to permit a shear per unit of gross area of six-tenths the permissible tension per unit of gross area, but the writer prefers five-tenths instead of six-tenths and is confirmed in this preference by the fact that ample webs are advisable for stiffness and reduce somewhat the magnitude of the errors involved in the ordinary theory of flexure. He also considers it best in important cases to compute the maximum shear by the theoretical formula given in textbooks on Mechanics* instead of proportioning for a mean intensity of shear considered as carried exclusively by the web. In rolled I-beams it does not make much difference, but for plate girders the maximum intensity of the shear in the web indicated by theory is ordinarily 10 to 25% greater than the mean indicated by the ordinary method.

That there are "faults in the theory of flexure" has been shown by Bach, Love, Saint Venant, and others, and by the writer in a paper,† thus entitled, and its discussion, to which references are made in this Part 8.

One of these faults is the neglect of the influence of shear on cross-sections, by reason of which originally plane cross-sections do not remain plane during flexure (as ordinarily assumed), but are forced into reversed curves somewhat like a long $\int$ only much less pronounced, as illustrated in Figs. 3‡ and 4§ of the writer's paper previously mentioned.

In consideration of the fact that because of the fault cited the ordinary theory of flexure indicates for the extreme fibers intensities of stress that are too small, an approximate method was devised by the writer‖ and improved by J. P. J. Williams, M. Am. Soc. C. E.,¶ for computing the percentage, P_e, which on account of this fault should, theoretically, be added to the intensity of the stress as computed from the ordinary theory. This method has been adapted to the purposes of this paper.

For steel beams of rectangular cross-section uniformly loaded and simply supported**:

$$P_e = 8.88 \frac{d^2}{x\,(l - x)} \qquad (19)$$

* It is given in the Carnegie Pocket Companion, 1913 Edition, p. 180, and 1923 Edition, p. 151.

† *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), pp. 895–981.

‡ *Loc. cit.*, p. 899.

§ *Loc. cit.*, p. 901.

‖ *Loc. cit.*, pp. 896–897.

¶ *Loc. cit.*, pp. 932–953.

** *Loc. cit.*, p. 940.

in which, l, is the length; d, the depth between extreme fibers; and x, the distance from the end to the point at which the percentage to be added, P_e, is desired.

TABLE 17.—LARGE BUILT PIN-ENDED COLUMNS, LOADED BEYOND THEIR USEFUL LIMIT POINTS.

R = ratio of plastic to elastic deformation at the U. L. P.
L = load which caused plastic deformation equal to one-third the elastic deformation. After removal of load, plastic deformation becomes permanent set.

Item.	Pages*.	Description.	$\frac{l}{r}$	U. L. P., in thousands of pounds per square inch.	R, approximate percentage.	L: In thousands of pounds per square inch.	L: Percentage of U. L. P.	MAXIMUM: In thousands of pounds per square inch.	MAXIMUM: Percentage of U. L. P.
1	23, 64	Nickel steel	25	49.0	15	54.0	110	61.7	126
2	26, 65	" "	37	52.0	14	58.5	113	61.4	118
3	32, 67	Mayari steel	43.5	†	....	¶	....	61.5	....
4	42, 73	" "	38.5	52.0	12	¶	....	64.7	124
5	54, 79	" "	27.7	‡	....	¶	....	47.2	....
6	58, 81	" "	34.5	‡	....	¶	....	49.4	....
7	62, 83	" "	27.1	§	....	¶	....	57.7	....
8	36, 69	Silicon steel	43.5	39.0	17	42.0	108	52.8	135
9	46, 75	" "	38.5	41.0	21	44.0	107	51.7	126
10	38, 71	Chrome steel	43.5	35.0	19	38.0	109	49.2	141
11	48, 76	" "	38.5	36.0	22	40.3	112	51.8	144
12	29	Carbon steel	43.5	30.0	20	32.6	109	38.4	128
13	34, 68	" "	43.5	31.0	21	34.5	111	41.7	135
14	40, 72	" "	38.5	‖	....	¶	....	45.3	....
15	44, 74	" "	38.5	33.0	20	37.0	112	46.6	141
16	52, 78	" "	28.7	28.5	17	¶	....	34.3	120
17	56, 80	" "	36.1	27.0	19	30.0	111	31.0	115
18	60, 82	" "	27.6	23.0	22	26.0	113	32.4	141

* This column refers to pages in *Technologic Paper No. 101*, U. S. Bureau of Standards. Additional information regarding these columns is given in Table 18. The term, U. L. P., does not appear in connection with these tests in *Technologic Paper No. 101*, but it was scaled by the writer from graphs or determined from data given on the pages cited.

† Exceeds 47.0, which was the load at the last measured deformation, and, from the general trend of the curve is, probably 50.

‡ Had not been reached at 45.0. Photographs of column after failure show that failure was due to insufficient stiffness at forked ends.

§ Greater than 50.0.

‖ Greater than 35.0.

¶ Deformations indicative of L were not observed.

By substituting different constant factors in place of 8.88 (Equation 19), it can be applied to I-shaped beams and plate girders.

Substituting C_e, a factor constant for similar cross-sections, in place of 8.88,

$$P_e = C_e \frac{d^2}{x\,(l - x)} \qquad \text{(20)}$$

At the center of the beam or girder,

$$P_e = \frac{4\,C_e\,d^2}{l^2} \qquad \text{(21)}$$

For instance, if $C_e = 10$, $d = 30$ in., and $l = 80$ in., the percentage to be added is 5.625.

TABLE 18.—Tests of Large Steel Columns.

(Made by J. H. Griffith, Associate Engineer Physicist, and J. G. Bragg, Associate Assistant Physicist. Compiled from *Technologic Paper No. 101*, U. S. Bureau of Standards.)

The U. L. P.'s are not stated in *Technologic Paper No. 101*, U. S. Bureau of Standards, but have been taken from the stress-strain diagrams for tests where the diagrams reached this point. The ratios, $\frac{l}{r}$, are different for different axes. The greatest ratio is used. See *Technologic Paper No. 101*, p. 22, for chemical analyses and maximum tensile stress for specimen; pages 20 and 21 for cross-sections; and the pages cited in first column for the remaining data. The columns in each group had nominally identical cross-sections.

Page.*	Name of steel.	Average Chemical Composition, Percentage.								Cross-Section.			U. L. P., in pounds per square inch.	Maximum load stress, in pounds per square inch.	Maximum tensile stress for specimen, in pounds per square inch.	$\frac{l}{r}$
		Ni.	C.	P.	Mn.	S.	Si.	Cr.	Cu.	Area, in square inches.	Thickest pieces.	Thinnest pieces.				
23	Nickel.....	3.51	0.29	0.014	0.59	0.031	....	....	....	88.8	11/16-in. plate	1/4-in. angle	49 000	61 700	93 300	25
26	Nickel.....	3.59	0.32	0.015	0.59	0.033	....	....	....	110.3	29/32-in. plate	1/4-in. angle	52 000	61 400	93 100	37
29	Carbon....	....	0.36	0.012	0.43	0.030	....	....	....	74.6	1-in. plate	3/8 in. angle	30 000	38 400	76 700	43.5
34	Carbon....	....	0.28	0.017	0.64	0.031	....	....	....	76.9	1-in. plate	3/8-in. angle	31 000	41 700	73 400	43.5
32	Mayari....	1.12	0.34	0.010	0.68	0.037	0.10	0.53	0.10	74.9	1 in. plate	3/8-in. angle	†	61 500	100 600	43.5
36	Silicon	....	0.35	0.017	0.83	0.039	0.38	....	....	76.9	1-in. plate	3/8-in. angle	39 000	52 800	87 700	43.5
38	Chrome...	....	0.31	0.017	0.49	0.028	0.13	0.66	....	76.9	1-in. plate	3/8-in. angle	35 000	49 200	82 700	43.5
40	Carbon....	....	0.36	0.012	0.40	0.030	....	....	....	75.4	11/16-in. plate	3/8-in. angle	†	45 300	75 900	38.5
44	Carbon....	....	0.28	0.017	0.64	0.031	....	....	....	77.2	11/16-in. plate	3/8-in. angle	33 000	46 600	74 400	38.5
42	Mayari....	1.12	0.34	0.010	0.68	0.037	0.10	0.53	0.10	75.3	11/16-in. plate	3/8-in. angle	52 000	64 700	101 200	38.5
46	Silicon	....	0.35	0.017	0.83	0.039	0.38	....	....	77.2	11/16-in. plate	3/8-in. angle	41 000	51 700	88 300	38.5
48	Chrome...	....	0.31	0.017	0.49	0.028	0.13	0.66	....	77.2	11/16-in. plate	3/8-in. angle	36 000	51 800	83 600	38.5
52	Carbon....	....	0.23	0.012	0.49	0.027	....	....	....	56.0	7/16-in. plate	1/4-in. plate	†	34 300	65 400	28.7
54	Mayari....	1.58	0.33	0.016	0.64	0.027	....	0.37	....	41.7	7/16-in. plate	3/8-in. angle	†	47 200	99 800	27.7
56	Carbon....	....	0.23	0.012	0.49	0.029	....	....	....	95.8	11/16-in. plate	1/4-in. plate	27 000	31 000	65 400	36.1
58	Mayari....	1.58	0.33	0.016	0.64	0.027	....	0.37	....	66.0	17/32-in. plate	1/4-in. plate	†	49 400	99 800	34.5
60	Carbon....	....	0.23	0.012	0.49	0.029	....	....	....	118.2	3/4-in. plate	1/2-in. angle	23 000	32 400	65 400	27.6
62	Mayari....	1.58	0.33	0.016	0.64	0.027	....	0.37	....	76.5	19/32-in. plate	1/2-in. angle	†	57 700	99 800	27.1

* *Technologic Paper No. 101*, U. S. Bureau of Standards.
† Shortening in gauged length not measured at point high enough in stress to indicate the U. L. P.

When the load instead of being distributed from end to end is confined within and uniformly distributed over a distance, $2a$, at the center, the percentage of increase at the center* is:

$$P_e = \frac{C_e\, d^2}{a\,(l - a)} \qquad \text{(22)}$$

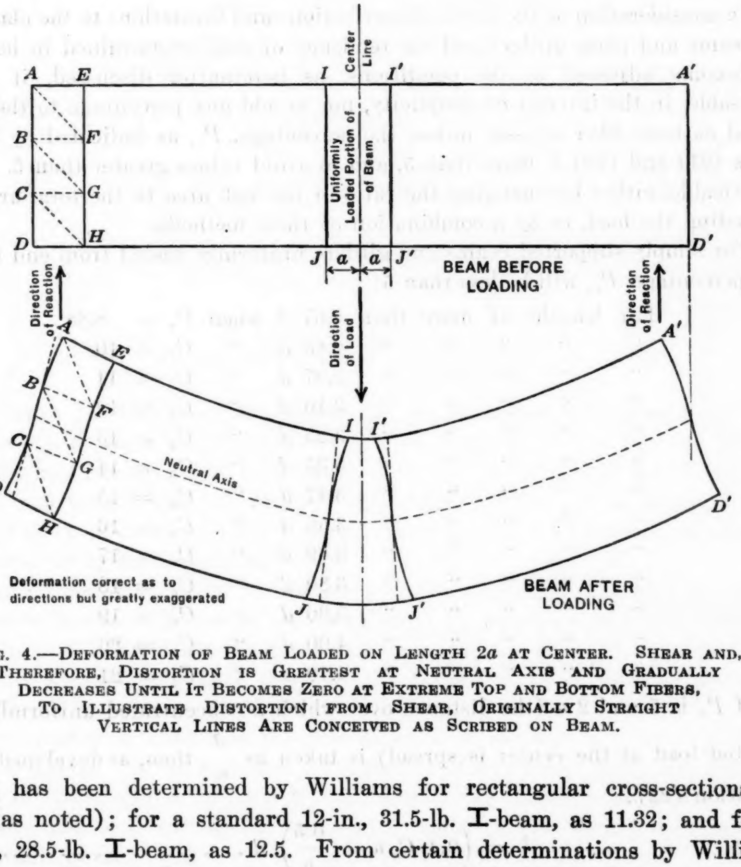

FIG. 4.—DEFORMATION OF BEAM LOADED ON LENGTH $2a$ AT CENTER. SHEAR AND, THEREFORE, DISTORTION IS GREATEST AT NEUTRAL AXIS AND GRADUALLY DECREASES UNTIL IT BECOMES ZERO AT EXTREME TOP AND BOTTOM FIBERS. TO ILLUSTRATE DISTORTION FROM SHEAR, ORIGINALLY STRAIGHT VERTICAL LINES ARE CONCEIVED AS SCRIBED ON BEAM.

C_e has been determined by Williams for rectangular cross-sections as 8.88 (as noted); for a standard 12-in., 31.5-lb. I-beam, as 11.32; and for a 12-in., 28.5-lb. I-beam, as 12.5. From certain determinations by Williams and from others by the writer, the following empirical rules for obtaining the approximate value of C_e for plate girders and I-beams have been devised.

Let,

"Ratio" = the ratio of web cross-sectional area to total cross-sectional area, considering the web as of full depth.

Then, for I-beams:

$$C_e = \frac{1.96}{\text{Ratio}} + 7.0$$

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* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 952.

For plate girders:

With "Ratio" between 0.28 and 0.36.... $C_e = \dfrac{7.17}{\text{Ratio}} - 4.7$

With "Ratio" between 0.36 and 0.54.... $C_e = \dfrac{3.32}{\text{Ratio}} + 6.0$

With "Ratio" between 0.54 and 0.71.... $C_e = \dfrac{5.72}{\text{Ratio}} + 1.6$

In consideration of the initial imperfections and limitations to the elasticity of beams and plate girders and the tendency of steel overstrained in bending to become adjusted to the conditions, as hereinafter discussed, it seems advisable, in the interest of simplicity, not to add any percentage to the computed extreme fiber stresses unless the percentage, P_e, as indicated by Equations (21) and (22) is more than 5, and to avoid values greater than 5, where practicable, either by changing the ratio of the web area to the total area, by spreading the load, or by a combination of these methods.

For simply supported beams and girders uniformly loaded from end to end the percentage, P_e, will be less than 5:

For lengths of more than 2.67 d when $C_e = 8.88$
" " " " " 2.83 d " $C_e = 10$
" " " " " 2.97 d " $C_e = 11$
" " " " " 3.10 d " $C_e = 12$
" " " " " 3.23 d " $C_e = 13$
" " " " " 3.35 d " $C_e = 14$
" " " " " 3.47 d " $C_e = 15$
" " " " " 3.58 d " $C_e = 16$
" " " " " 3.69 d " $C_e = 17$
" " " " " 3.80 d " $C_e = 18$
" " " " " 3.90 d " $C_e = 19$
" " " " " 4.00 d " $C_e = 20$
" " " " " 4.10 d " $C_e = 21$

If P_e is 5 and $2a$ (the distance over which a concentrated uniformly distributed load at the center is spread) is taken as $\dfrac{d}{n}$, then, as developed from Equation (22):

$$l = \left(0.4\, C_e\, n + \frac{0.5}{n}\right) d \qquad \text{(23)}$$

For simply supported beams and girders, uniformly loaded over a distance or spread of $\dfrac{d}{n}$, the percentage, P_e, will be less than 5:

For lengths of more than $\left(3.55\, n + \dfrac{0.5}{n}\right) d$, when $C_e = 8.88$

" " " " " $\left(4.0\, n + \dfrac{0.5}{n}\right) d$, " $C_e = 10$

" " " " " $\left(4.4\, n + \dfrac{0.5}{n}\right) d$, " $C_e = 11$

For lengths of more than $\left(4.8\ n + \frac{0.5}{n}\right) d$, when $C_e = 12$

" " " " " $\left(5.2\ n + \frac{0.5}{n}\right) d$, " $C_e = 13$

" " " " " $\left(5.6\ n + \frac{0.5}{n}\right) d$, " $C_e = 14$

" " " " " $\left(6.0\ n + \frac{0.5}{n}\right) d$, " $C_e = 15$

" " " " " $\left(6.4\ n + \frac{0.5}{n}\right) d$, " $C_e = 16$

" " " " " $\left(6.8\ n + \frac{0.5}{n}\right) d$, " $C_e = 17$

" " " " " $\left(7.2\ n + \frac{0.5}{n}\right) d$, " $C_e = 18$

" " " " " $\left(7.6\ n + \frac{0.5}{n}\right) d$, " $C_e = 19$

" " " " " $\left(8.0\ n + \frac{0.5}{n}\right) d$, " $C_e = 20$

" " " " " $\left(8.4\ n + \frac{0.5}{n}\right) d$, " $C_e = 21$

By care in designing, the deficiency in the computed intensity of the extreme fiber stress can be reduced to less than 5% in all but very short I-beams and girders. For instance, if $C_e = 20$, $d = 24$, and $2\,a = 8$, then, for $P_e = 5$, $n = 3$, and $l = 58$. If l is less than 58 but not too small, P_e can be kept down to 5 or less by increasing either $2\,a$, or the ratio of web area to total area, or by both methods.

The values of P_e given by Equations (20), (21), and (22) are too large. The error increases as the indicated value of P_e increases, but is very slight up to $P_e = 10$; however, when these equations indicate P_e as 50, P_e is probably not more than 35; and when they indicate P_e as 100, P_e is probably not more than 50.

The manner of spreading the load depends on the way it is applied to the beam or girder. If the web has a surplus of cross-section above the requirements of shear and is sufficiently stiff, and if the load is conveyed to it by adequate means, it will spread the load to some extent beyond the limits of the actual contact.

The ordinary theory of flexure is developed on the tacit assumption that the loads and reactions are applied to beams and girders so that each increment of each horizontal layer, at or vertically in line with each increment of load, receives directly just that share of the load increment necessary to produce the theoretical changes in its shear without any part having been conveyed to it by compression or tension through adjacent layers. This tacit assumption is sometimes approximated, but is never fully realized; consequently, vertical stresses in the web occur at, and inclined stresses in the web originate at, points of loading and reaction, and these inclined stresses have horizontal components, all of which affect the web and flanges although they are neglected

in the ordinary theory of flexure. These uncomputed stresses, especially if the load is concentrated, make it advisable to provide web section and web reinforcement in excess of the requirements of ordinary theory, and, in girders with flange cover-plates, to require that cover-plates of the bottom flange extend beyond the nominal theoretical limits when the load is applied at the top and similarly for the top flange when the load is applied at the bottom.

Web section and reinforcement in excess of the requirements of ordinary theory may reasonably be expected to mitigate the effects of concentration much the same as would an increase in the length of direct application.

The theory of flexure is based on ideal material perfectly elastic within limits, but the theory stops at these limits. The effects of the differences between ideal steel and the actual steel of commerce as applied to beams and plate girders under transverse loads require special consideration.

Rolled I-beams are peculiarly liable to auto-stress* and to variations in the elastic resistances of their different parts. The U. L. P.'s in pounds per square inch, as scaled from the diagrams of tests of specimens from the webs, flanges, and roots of certain I-beams tested by the late Edgar Marburg, M. Am. Soc. C. E.†, are, as given in Table 19.

TABLE 19.—SPECIMEN TESTS FROM WEB, FLANGE, AND ROOT OF LARGE I-BEAMS.

Beam.	Web, U. L. P., in pounds per square inch.	Flange, U. L. P., in pounds per square inch.	Root, U. L. P., in pounds per square inch.
15-in., 38-lb. I-beam	42 500	......	26 000
24-in., 72-lb. I-beam	46 000	36 000	38 000
24-in., 80-lb. Standard I-beam	30 500	28 400	25 000
30-in., 120-lb. I-beam	37 000	31 500	30 100
15-in., 73-lb. Girder beam	39 000	......	22 000
24-in., 120-lb. Girder beam	36 400	33 300	28 000
30-in., 175-lb. Girder beam	34 900	28 000	27 000

Fatigue tests of rotating shafts, of which there have been many (some are cited in Table 5), and fatigue tests by Wöhler and others of presumedly square or rectangular bars loaded alternately in opposite directions, show, by inference, that fields of seemingly perfect elasticity can be developed by alternate bending in opposite directions; and fatigue tests by Wöhler and others, presumedly on square or rectangular bars, under loads which in each case are in the same direction but repeatedly vary between a given minimum and maximum, show that these fields of seemingly perfect elasticity can be shifted so that the stress in the extreme fiber at the top of the beam varies between computed elastic limits both of which are in compression whereas the stress in the extreme fiber at the bottom of the beam varies between computed elastic limits both of which are in tension.

It has been shown in Parts 5 and 6 that for compression and tension members, loading which would perfect the elasticity to a point much above the

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), pp. 913 and 931.

† Reported in "Tests of Standard I-Beams and Bethlehem Special I-Beams, and Girder Beams", *Proceedings*, Am. Soc. for Testing Materials, Vol. IX (1909), p. 378.

U. L. P. is not permissible because there the yield develops so rapidly that a slightly greater load would cause seriously detrimental extension in tension members or shortening in compression members. In beams and plate girders under transverse loads, the plastic deflection develops more gradually than the plastic deformation in tension and compression members; hence if the beams are held against lateral deflection and the metal is not so attenuated as to permit buckling or crippling of the web or flanges, there is greater opportunity for shifting the elastic field. The difference in this respect between the plastic extension of a soft steel eye-bar and the plastic deflection of beams under transverse loads is illustrated by the following comparison:

(*a*).—An eye-bar tested at the Watertown Arsenal* had a U. L. P. of 29 000 lb. per sq. in. The plastic deformation at this point was quite small and of little consequence *per se,* but the rate of extension indicated that a pronounced yield had set in. Between 31 000 and 32 000 lb. per sq. in. there was a definite yield point and at about 20% above the yield point the plastic extension or permanent set became more than sixteen times as great as the computed elastic extension.

(*b*).—Twenty 15-in. I-beams of 21-ft. span and nominally equivalent strength were tested at Ambridge, Pa., loaded at the center†. If in each case these loads had been 36 600 lb. (including one-half the weight of the beam considered as concentrated at the center), and the beams had been perfectly elastic and simply supported, the computed stress in the extreme fiber by the ordinary theory of flexure would have been 39 000 lb. per sq. in., which is about the average yield point indicated by the specimen tests. The actual loads which produced permanent sets of 0.4 in. were all within 7% of their average, which average was 65.7% greater than the load of 36 600 lb. per sq. in., while the permanent set was only a little more than one-fourth of the computed elastic deflection. The beams, however, were not "simply supported" as the end connections and supports must have exercised some restraining influence. If this restraining influence amounted to a bending moment at each end of one-third the moment at the center (an exaggerated proportion), the loads which produced the permanent set of 0.4 in. even in that case would have been more than 20% in excess of the 36 600 lb. per sq. in. and the permanent set would not have been much more than one-half the computed elastic deflection.

When I-beams and plate girders, in which the metal is not so attenuated as to permit buckling or crippling of web or flanges, have properly designed details, are adequately proportioned for shear, are not too short, and have their flanges held against lateral deflection, it seems reasonable from the foregoing analysis to assume that their utilizable capacity in each case will be as great as the load for which the intensity of the stress in the extreme fiber, computed from the ordinary theory of flexure, equals u, u being the useful limit point of the flange similarly as described in the rule in Part 5 (page 1758), for tension and compression members. It is probable, for I-beams at least, that the utilizable capacity is somewhat greater than this when the load varies between

* *Transactions,* Am. Soc. C. E., Vol. LXXV (1912), p. 968.

† *Loc. cit.,* pp. 917–918.

a minimum and maximum of reduced range, provided the minimum is not too close to the maximum. If the minimum is too high it does not afford the plastic metal sufficient opportunity to recover from elastic fatigue but causes the plastic deformation to increase gradually until failure results. A very slow increase in plastic deflection of beams may continue for a long time under heavy static loads, as shown in the case of a pair of 15-in., 50-lb. I-beams.*

Part 9.—Combined Compression and Flexure

When in addition to the direct load a column is subjected to intentional flexure, it follows from the assumptions in Part 6 that the utilizable capacity is determined by one of three following limitations, *A*, *B*, or *C*, whichever gives the smallest capacity:

Limitation A.—The utilizable capacity shall not be considered greater than indicated by Equation (1) for columns subjected to direct load without intentional flexure.

Limitation B.—The greatest intensity of stress, *f*, from direct load, intentional flexure, and unintentional eccentricity (assumed as in Part 6 as $\frac{l}{700}$ in the form of a sinusoidal bow) shall not be greater than *u*.

Let,

f_1 = maximum intensity of intentional primary flexure.
f_2 = maximum intensity of unintentional primary flexure.

$$f_c = f_1 + f_2 \quad \text{.......................} (24)$$

From Equation (3):

$$f = p + f_c + f_c\left(\frac{p}{q - p}\right) \quad \text{..................} (25)$$

Equation (25) is exact only when the moment diagram is a sinusoid; but it is a close approximation for all cases in which the maximum bending moment occurs at or near the center of the column. For computing f_2, *e*, as, in Part 6, may be taken as $\frac{l}{700}$ and $\frac{v}{r}$ as 1.68. Then, from analysis similar to that by which Equation (8) was developed,

$$f_2 = 0.00240\, p\, \frac{l}{r} \quad \text{.......................} (26)$$

The direct load per unit of area, *p*, which in combination with f_1 and f_2, will produce a stress intensity, *f*, equal to *u*, can be found from Equation (25) by first assuming *p* and then making successive trials, but this tedious process is unnecessary as it can be approximated with sufficient closeness from the following empirical formula:

$$p = c - f_1 \quad \text{.......................} (27)$$

c having been first found from Equation (11).

To show the approximate accuracy of Equation (27), *f* has been determined in Table 20 (*a*) for a large range of cases from Equation (25) using for *p* the value obtained from Equation (27). If the value obtained for *p* had been

* *Transactions*, Am. Soc. C. E., Vol. LXXV (1912), p. 975.

exact, f in all cases would have been the same as u. It differs from u by less than 2% for all cases except those in bold face type.

TABLE 20.—JUSTIFICATION FOR EMPIRICAL FORMULA EQUATION (27).

(For this comparison, columns in which the ratio, $\frac{v}{r} = 1.68$, were chosen. For smaller ratios the error would be less.)

(*a*) MAXIMUM STRESS INTENSITY, f, INDICATED BY EQUATION (25) FROM USING THEREIN THE VALUE FOR p INDICATED BY EMPIRICAL EQUATION (27)*.

Criterion u, in pounds per square inch.	$\frac{l}{r}$	When $f = 0.1\,p$, in pounds per square inch.	When $f = 0.2\,p$, in pounds per square inch.	When $f = 0.5\,p$, in pounds per square inch.	When $f = p$, in pounds per square inch.
20 071	20	20 098	20 054	19 935	19 791
20 071	40	20 047	19 996	19 811	**19 530**
20 071	70	19 991	19 685	**19 631**	**19 069**
20 071	120	20 258	19 873	See (*b*)	See (*b*)
35 105	20	35 822	35 278	35 121	34 877
35 105	35	35 248	35 251	35 071	34 646
35 105	60	35 055	35 153	34 828	**33 888**
35 105	90	35 115	34 750	See (*b*)	See (*b*)
50 063	20	50 600	50 588	50 429	50 095
50 063	35	50 457	50 599	50 499	49 884
50 063	50	50 209	50 522	50 358	49 225
50 063	70	50 230	50 311	See (*b*)	See (*b*)

(*b*) COMPARISONS OF VALUE OF p INDICATED BY EMPIRICAL EQUATION (27) AND RATIONAL EQUATION (31).

Criterion u, in pounds per square inch.	$\frac{l}{r}$	When $f = 0.5\,p$.		When $f = p$.	
		p as per Equation (31), in pounds per square inch.	p as per Equation (27), in pounds per square inch.	p as per Equation (31), in pounds per square inch.	p as per Equation (27), in pounds per square inch.
20 071	120	8 762	8 104†	6 298	6 078†
35 105	90	13 781	14 996	9 430	11 247
50 063	70	20 127	23 467	13 206	17 600

* The value of f as indicated in (*a*) differs from u by less than 2% in all cases except those recorded in bold-face type.

† For these two cases and for these two only, Equation (27) seems to be critical; in all the other cases Equation (31) seems to be critical; actually, Equation (31) is critical in all the cases. The use of Equation (27) for the two cases in which it seems to be critical is conservative, and the error involved on the side of safety is not excessive.

Limitation C.—The deflection under combined intentional and unintentional bending moments shall not exceed $\frac{l}{300}$.

Let,

M_1 = the intentional primary bending moment;

Δ = the deflection;

A = the cross-sectional area; and,

e_1 = the primary intentional eccentricity.

From the theory of flexure:

$$\Delta = \frac{(\text{Bending moment})\ l^2}{C_2\ E\ A\ r^2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (28)$$

in which, for beams or columns of constant cross-section with frictionless hinged ends, C_2 equals 9.6, for a uniformly distributed transverse load; 12.0 for a single transverse load concentrated at the middle; or 8.0, when the bending moment is constant from end to end; or π^2, when the moment diagram is a sinusoid.

For combined compression and flexure with $\Delta = \frac{l}{300}$, and unintentional eccentricity $= \frac{l}{700}$,

$$\frac{l}{300} = \frac{\frac{\pi^2}{C_2}\ M_1\ l^2}{\pi^2\ E\ A\ r^2} + \frac{p\ A\ \frac{l}{700}\ l^2}{\pi^2\ E\ A\ r^2} + \frac{p\ A\ \frac{l}{300}\ l^2}{\pi^2\ E\ A\ r^2} \ldots\ldots\ldots\ldots (29)$$

Substituting q (Euler's formula) for its value, as given in Part 6, and reducing,

$$p = 0.7\ q - \frac{210\ \frac{\pi^2}{C_2}\ M_1}{A\ l} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (30)$$

When the primary bending moment is caused by an intentional eccentricity, e_1, constant from end to end, $M_1 = p\ A\ e_1$ and $C_2 = 8$; whence, by substitution in Equation (30),

$$p = \frac{0.7\ q}{1 + \frac{259.1\ e_1}{l}} \ldots\ldots\ldots\ldots\ldots\ldots\ldots (31)$$

In some cases the value of p, as determined from Equation (31), is somewhat greater than indicated by the empirical Equation (27); but notwithstanding this is slightly less than indicated by Equation (25). In such cases, to be precise, Equation (31) should govern, but this fact can only be ascertained by a tedious determination of p by trial from Equation (25). The error involved in using Equation (27) in such cases is not excessive and is on the side of safety, as shown in Table 20 (*b*).

When the maximum possible value of p within the utilizable capacity approaches or is less than one-half u, it indicates that there is liable to be tension in the convex side of the column.

The compression chords of bridges being usually continuous, it may be advisable in important cases when the transverse load is very heavy to treat them rigidly as continuous girders; but ordinarily the only transverse load on a column is its own weight, and for this case, if the column has riveted connections to adjacent members of sufficient stiffness to exercise a fair restraint on its ends, the l of the tables and formulas may be taken as from three-fourths to two-thirds of the length of the column.

In proportioning for combined compression and flexure the only alternative, except for a few particular cases, is to rely on analysis or guesswork, as there

are not enough experiments to indicate general empirical formulas; but the Watertown Arsenal Reports record tests of four full-sized, pin-ended, eccentrically loaded, built columns, and one of a pin-ended, steel tube column subjected to longitudinal and transverse loading, which tests give an opportunity for comparing actual and theoretical results. This opportunity has been utilized by compiling Table 21. The best means of comparison is by deflections.

Let,

Δ_p = the deflection from primary bending moment.

This is readily obtained by the theory of flexure (see text following Equation (28)).

$$\Delta = \Delta_p + \frac{p\ A\ \Delta_p\ l^2}{\pi^2\ E\ A\ r} \qquad (32)$$

By substituting q for its value, $\frac{l^2}{\pi^2\ E\ r^2}$, and reducing,

$$\Delta = \Delta_p \left(1 + \frac{p}{q - p}\right) \qquad (33)$$

This use of the factor, π^2, is not rigid analysis, but for equal eccentricity at both ends and to the same side of the axis and for any symmetrical transverse loading its use in Equation (32) gives results for frictionless hinged-ended columns which for all ordinary cases differ from results of rigid analysis by less than one-half of 1 per cent.

Actually columns under combined compression and flexure are subject to friction on their pins, to unintentional eccentricity, and, as failure is approached, to considerable plastic deformation, none of which is allowed for in Equation (32). The effect of friction is to hinder deflection until the bending moment is sufficient to turn the ends of the column on the pin; plastic deformation increases deflection; and unintentional eccentricity, according to its direction, may either increase or decrease deflection. In these circumstances, the tests are not conclusive as to the accuracy of the theory and assumptions used, but, intelligently compared, the tests are strong evidence of the safety and general soundness of the theory.

Part 10.—Combined Tension and Flexure

An analysis of flexure combined with tension is not as important as that of flexure combined with compression for the reason that the bending moment from direct tension reduces the total stress intensity, whereas in the case of direct compression it increases the total stress intensity.

Let,

p_t = the intensity of the direct tension.

Then, for a tension member with frictionless hinged ends, by analysis analogous to that by which Equations (3) and (25) were developed,

$$f = p_t + f_c - f_c \left(\frac{p}{p - q}\right) \qquad (34)$$

When the tension member is loaded to its utilizable capacity, $f = u$, and,

$$p_t = u - f_c + f_c \left(\frac{p}{p - q}\right) \qquad (35)$$

TABLE 21.—TESTS OF PIN-ENDED COLUMNS ECCENTRICALLY LOADED; AND FLAT-ENDED COLUMN WITH COMBINED LONGITUDINAL AND TRANSVERSE LOADS.

(Compiled from the Watertown Arsenal Reports for 1883, 1884, and 1909.)

NOTATION: p = longitudinal load per square inch; f = computed extreme fiber compression per square inch on the concave side of the column from longitudinal load and flexure; Δ_c = computed deflection in inches, equation (33); Δ_a = actual deflection; l = length in inches; r = radius of gyration in inches; I = moment of inertia, in inches⁴; v = distance, in inches, from the center of gravity to the extreme fiber on the concave side of the column; u = U. L. P. of material on the concave side of the column, as gauged by full-sized tests of short columns of similar 10-in. channels, 8-in. channels, and 5-in. lap-welded steel pipes.
The columns were subjected during the tests to friction from pins; they were liable to unintentional eccentricity; and, as failure was approached, they underwent some plastic deformation, none of which was considered in making the computations tabulated.

FOR WROUGHT-IRON COLUMNS WITH TOP COVER-PLATE AND BOTTOM LATTICE-BARS, E = 28 000 000 LB. PER SQ. IN.																FOR STEEL, E = 30 000 000 LB. PER SQ. IN.			
Test 1632; year, 1883, p. 167; two 10-in. channels; one cover-plate, 13 by 44 in.; actual area, 17.57 sq. in.; l = 300 in.; r = 4.01 in.; $\frac{l}{r}$ = 74.8 in.; eccentric application of load = 1.62 in.; I = 282.9 in.⁴; v = 6.62 in.; u = 31 000 lb. per sq. in.				Test 1633; year, 1883, p. 168; two 10-in. channels; one cover-plate, 13 by 44 in.; actual area, 17.721 sq. in.; l = 300 in.; r = 4.01 in.; $\frac{l}{r}$ = 74.8 in.; eccentric application of load = 1.64 in.; I = 285.4 in.⁴; v = 6.64 in.; u = 31 000 lb. per sq. in.				Test 350; year, 1884, p. 54; two 8-in. channels; one cover-plate, 12 by 43 in.; actual area, 12.48 sq. in.; l = 240.2 in.; r = 3.13 in.; $\frac{l}{r}$ = 76.7 in.; eccentric application of load = 1.72 in.; I = 122.5 in.⁴; v = 5.75 in.; u = 32 000 lb. per sq. in.				Test 351; year, 1884, p. 55; two 8-in. channels, 7.5 sq. in.; one cover-plate, 12 by 39 in.; 4.68; computed area,† 12.18 sq. in.; l = 240.2 in.; r = 3.14 in.; $\frac{l}{r}$ = 76.5 in.; eccentric application of load = 1.62 in.; I = 120.3 in.⁴; v = 5.65 in.; u = 32 000 lb. per sq. in.				Test 2087; year, 1909, p. 865; 5-in. lap-welded pipe, 0.36 in. thick; actual area, 5.255 sq. in.; l = 230 in.; r = 1.647 in.; $\frac{l}{r}$ = 139.5 in.; transverse uniform load 332 lb.; transverse center load, 1 482 lb.; I = 14.25 in.; v = 2.5 in.⁴; u = 31 000 lb. per sq. in.			
p.	f.	Δ_c.	Δ_a.	p.	f.	Δ_c.	Δ_a.	p.	f.	Δ_c.	Δ_a.	p.	f.	Δ_c.	Δ_a.	p.	f.	Δ_c.	Δ_a.
569	......	0.02	0.00	564	......	0.03	0.00	801	......	0.04	0.00	821	......	0.04	0.00	0	1 662	1.00	....
1 188	......	0.05	0.01	1 128	......	0.05	0.01	1 603	......	0.07	0.03	1 642	......	0.07	0.02	2 000	19 200	1.15	0.84
2 845	......	0.12	0.10	2 822	......	0.12	0.09	4 006	......	0.20	0.14	4 105	......	0.19	0.12	4 000	21 580	1.36	0.99
5 692	......	0.26	0.22	5 643	......	0.26	0.21	8 013	......	0.44	0.37	8 210	......	0.42	0.32	6 000	24 360	1.65	1.22
11 388	21 770	0.60	0.54	11 287	21 670	0.59	0.51	12 019	29 270	0.73	0.67	12 315	28 680	0.70	0.58	8 000	27 590	2.11	1.57
14 228	28 440	0.81	0.77	14 108	28 280	0.80	0.74	14 423	36 900	0.94	0.90	14 778	36 150	0.91	0.82	9 172*	29 850*	2.52	2.21
15 936	32 790	0.95	0.95	15 801	32 590	0.94	0.91	16 026	42 490	1.10	1.20	16 420	41 620	1.06	1.05	Load D	ropped		
17 075	35 840	1.05	1.15	16 930	35 630	1.04	1.20	16 242*	43 280*	1.12	1.40	17 093*	44 000*	1.13	1.38	8 000	......	2.28	2.42
17 416*	36 820*	1.09	1.50	17 825*	36 730*	1.08	1.38	......	......	....	....	......	......	....	....	LoadSu	stained		

* After reaching this maximum, the load dropped and the deflection increased.
† The amount given in the report as actual area was evidently erroneous.

These equations are strictly accurate only when the primary moment diagram is a sinusoid, but they give close approximations in other cases. They are similar to, or possibly identical with, equations now in use. Equation (35) can be solved only by trial, but in many cases the correction involved is worth the trouble.

In some special instances it may be advisable to treat the tension chords of bridges rigidly as continuous girders and, in other cases of tension members with riveted end connections, to take the l of the formulas and tables as from three-fourths to two-thirds of the length of the members; similarly (as outlined at the end of Part 9) for columns.

Part 11.—Variation of the Characteristic Useful Limit Point Between Different Members of Nominally the Same Grade of Steel

Table 22, listing tests of bars forged down from an open-hearth steel ingot in the direction of its length, shows many variations—in tensile strength between 104 800 and 113 000 lb. per sq. in., in U. L. P.'s between 46 000 and 72 000 lb. per sq. in., and in ratios of U. L. P.'s to tensile strength between 42.4 and 65.5 per cent. In this series the range of the ratio of the U. L. P. to the tensile strength is great but not extreme: For example, Table 7 cites a case where this ratio was 84.2%; the weighted average ratio of U. L. P.'s to tensile strength for twelve specimens representative of the extra heavy H-section columns in Table 11 was only 40.8% (with a ratio in one specimen from the end of the flange of 32.6%); and a specimen from the head of an electric furnace steel rail had a tensile strength of 116 600 lb. per sq. in., a U. L. P. of 47 000 lb. per sq. in., and a ratio of U. L. P. to tensile strength of only 40.3 per cent.*

The chief value of Table 22 is not in the ranges of the properties it shows but in the light it sheds on the fact that the reduction in cross-section and the temperature at which reduced are causes of variation between the U. L. P.'s of different specimens of the same steel. Differences in conditions under which shapes and plates cool after rolling, differences in auto-stresses produced in cooling, and other occurrences during manufacture are, probably, further causes of differences between the U. L. P.'s of different specimens.

The specifications for "Structural Steel for Bridges" (not including eyebars and rivets) of the American Society for Testing Materials and of the American Society of Civil Engineers† permit a variation in tensile strength of 5 000 lb. either way from 60 000 lb. per sq. in., without any corresponding variation in working stresses. If representative specimens for one member of a structure with the lowest permissible tensile strength (55 000 lb. per sq. in.) have a U. L. P. of 40%, and other specimens of another member in the same structure with the highest permissible tensile strength (65 000 lb. per sq. in.) have a U. L. P. of 60% or more, the characteristic U. L. P.'s for the members

* Watertown Arsenal Report, 1909, Vol. 2, p. 681.

† Final Report of the Special Committee on Specifications for Bridge Design and Construction on Specifications for Design and Construction of Steel Railway Bridge Superstructure, *Transactions*, Am. Soc. C. E., Vol. LXXXVI (1923), p. 487; also Final Report on Specifications for Design and Construction of Steel Highway Bridge Superstructure, *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1291.

themselves, according to indications given by the rule in the latter part of Part 5, will vary between 18 700 lb. per sq. in. for the one member and 39 000 lb. per sq. in. for the other; which is quite a wide range for which to use a common unit working stress. The extremes in the characteristic U. L. P.'s of the column, made of 60 000-lb. (desired) steel, tested for the Special Committee on Steel Columns and Struts were 19 000 and 35 000 lb. per sq. in., as shown in Table 11.

TABLE 22.—TESTS OF BARS FORGED DOWN FROM AN OPEN-HEARTH STEEL INGOT IN THE DIRECTION OF ITS LENGTH, VARIOUS TEMPERATURES AND AMOUNTS OF REDUCTION.

(Compiled from Watertown Arsenal Report, 1909, Vol. 3, pp. 869-896. Diameter of specimens, 0.798 in.; sectional area, 0.50 sq. in.; gauged length, 6 in.)

No. of test.	Marks.	Reductions under hammer, percentage.	Approximate finishing temperature, in degrees Fahrenheit.	Useful limit point, in 1 000 lb. per sq. in.	Tensile strength in 1 000 lb. per sq. in.	Ratio of useful limit point to tensile strength, percentage.	Elongation, percentage.	Contraction of area, percentage.
8 658	13 *B*	6.0	1 400	64.0	108.0	59.2	14.6	23.0
8 645	13 *T*	9.2	1 400	67.0	109.8	61.0	17.0	25.2
8 657	12 *B*	16.1	1 400	70.0	111.8	62.6	16.1	29.4
8 643	11 *T*	18.9	1 400	72.0	110.0	65.5	13.1	18.6
8 644	12 *T*	32.2	1 400	70.0	111.4	62.8	16.0	29.4
8 648	3 *B*	2.3	1 600	56.0	105.0	53.3	7.1	9.2
8 634	2 *T*	7.2	1 600	53.0	108.0	49.1	9.1	11.6
8 646	1 *B*	11.5	1 600	61.0	109.8	55.6	13.1	16.2
8 647	2 *B*	27.2	1 600	60.0	110.2	54.4	12.6	16.2
8 633	1 *T*	28.4	1 600	69.0	112.6	61.3	15.5	18.6
8 637	5 *T*	2.5	1 800	48.0	104.8	45.8	7.2	9.2
8 636	4 *T*	5.6	1 800	50.0	108.0	46.3	7.6	9.2
8 650	5 *B*	8.0	1 800	50.0	108.4	46.1	9.5	11.6
8 649	4 *B*	14.7	1 800	57.0	113.0	50.4	8.6	9.2
8 635	3 *T*	28.2	1 800	61.0	112.8	54.5	11.3	14.0
8 655	10 *B*	3.6	2 000	48.0	105.8	45.4	7.8	9.2
8 656	11 *B*	6.1	2 000	48.0	108.4	44.3	12.0	11.6
8 642	10 *T*	10.3	2 000	49.0	108.4	45.2	9.8	11.6
8 654	9 *B*	13.6	2 000	51.0	108.2	47.1	12.3	14.0
8 641	9 *T*	26.5	2 000	53.0	110.4	48.0	13.3	16.2
8 640	8 *T*	31.6	2 000	56.0	111.2	50.4	14.0	25.2
8 638	6 *T*	4.3	2 200	46.0	108.4	42.4	10.5	11.6
8 653	8 *B*	14.5	2 200	49.0	110.6	44.3	12.3	14.0
8 652	7 *B*	21.8	2 200	48.0	109.8	43.7	14.0	23.0
8 639	7 *T*	27.0	2 200	51.0	111.0	45.9	15.1	27.0
8 651	6 *B*	29.6	2 200	53.0	113.0	46.9	16.0	27.4

In 1875, the President of the United States appointed a Board for the Testing of Wrought-Iron, Steel, etc., and this Board appointed a Committee, headed by Commander L. A. Beardsley, U. S. N., to investigate wrought-iron chain. The Committee, as a preliminary, investigated the properties of different kinds or makes of wrought iron. It found for iron of the same grade considerable variation in tensile strength and great variation in what it termed "elastic limit" and described as the first perceptible stretch. Its tests in this regard were conducted slowly and with great care and with the aid of a magni-

fying glass. After experimenting with material and labor, in a rolling mill loaned without charge for the purpose by its proprietor, the Committee succeeded in rolling nine sizes of bars, varying in diameter from 1 in. to 2 in., with nearly uniform tensile strength and "elastic limit". After making these tests and experiments the Committee made a report* an excerpt from which is as follows:

"* * * as important differences exist in the proportionate strength of different-sized bars made of the same material, which are due entirely to differences in the processes by which they are manufactured, and as the elimination or reduction of such differences would necessitate such a great and expensive change in the system by which the bars are produced that it is not probable that it will be often attempted, it is necessary that these differences should be taken into consideration when estimates of the strength of any structure in which rolled wrought iron of different sizes is introduced are made, and in all tables of strength based upon the strength of such bars."

It is more than forty-six years since the Committee made its report. During this time steel has about supplanted wrought iron as a structural material and great changes have been made in rolling mills. The writer does not undertake to discuss what further changes, if any, in mill practice affecting the matter of differences between the U. L. P.'s of specimens of nominally the same grade of steel are possible, practicable, or probable; but inasmuch as wide differences in characteristic U. L. P.'s exist, he believes that they should be taken into consideration in estimating the utilizable capacity of steel members of structures.

Part 12.—Limitation of the Useful Limit Point and Maximum Tensile Strength, as Criteria of Utilizable Capacity, to Steel of Good and Suitable Quality and Workmanship.

The U. L. P. and maximum tensile strength of specimens of steel are criteria of the utilizable capacity of the steel members of structures only when the quality of the steel in other respects is good and suitable and the fabrication into structural members is properly performed.

This paper does not define what constitutes good and suitable quality and proper fabrication (the opinion of engineers has fairly well crystallized on these points and on the tests, analyses, and inspection by which they are gauged); but it does emphasize the importance of good and suitable quality and workmanship.

Part 13.—Factor of Safety

An exact determination of the utilizable capacity of a steel member of a structure, if achieved, would indicate precisely the greatest load that could statically or repeatedly, or the greatest opposite loads that could alternately, be applied during the lifetime of the structure (as determined by other considerations) without causing failure, ruin, or seriously objectionable deformation (plastic, elastic, or combined plastic and elastic). The utilizable capacity can not, of course, be exactly determined, but it can be approximately deter-

* Report of U. S. Board for Testing Iron, Steel, etc., 1881, Vol. 1, p. 45.

mined, as outlined in this paper, from representative specimen tests of the steel. The ratio of the utilizable capacity thus determined to the working load on the member is the member's approximate factor of safety; and the smallest factor of safety of any of the critical members of a structure is the approximate factor of safety of the structure as a whole.

Under the present practice of purchasing steel for structures and of designing and fabricating the steel members, the different members may have factors of safety that vary greatly from each other and from what is considered amply sufficient. The factor or margin of safety, instead of being entirely selected by the intelligent judgment of designers, depends largely on circumstances of manufacture of which designers, as a class, are inadequately informed and of which they usually take little heed.

Would it not be well for steel manufacturers, structural fabricators, physical metallurgists, and structural engineers, considered collectively as "makers of steel members of structures", to investigate the question whether or not it is practicable and advisable to classify the various sizes of steel plates, shapes, and bars, of nominally the same grade of steel into such groups that the variations of the U. L. P. within each group would be much less than the present variation within nominally the same grade of steel embracing all sizes of plates, shapes, and bars; and thus to enable engineers to take into account in proportioning members the U. L. P.'s of the different groups according to the sizes used?

THE HEXAGONAL SLAB DESIGN OF CONCRETE PAVEMENT

By Lewis A. Perry,* Assoc. M. Am. Soc. C. E.

Synopsis

This paper records the results of experiments conducted by the writer to learn what relation the shape of a pavement slab has on its strength.

Although these experiments were made in a laboratory with miniature specimens, the condition of moment, stress, and manner of loading was such that the results may be assumed to be representative of the behavior of pavement slabs in actual use. The uniform behavior of the test specimens lends weight to this assumption.

With the record of these experiments, a study of moments, as developed in various shaped slabs, is reported. This study is confirmed by the results of the experiments and presents some pertinent facts apparently hitherto unrecognized or considered unimportant.

The paper also discusses current practice in the light of information gained from these studies. Pavements of correct slab shape, as determined from these studies, are illustrated and described. Finally, the writer has attempted to define the limitations and essential features of an economic design.

Corner Weakness of Four-Sided Slabs.—A general failure of a concrete pavement is invariably a progressive destruction following corner fractures developed under flexural stress. Such corners may be right-angled corners at the intersection of joints, or joint with edge, or may be corners formed at the intersection of temperature cracks in an over-sized slab.

Proof that the 90° corner is susceptible to fracture under loading that is safe for the mid-portion of the slab is found in a study of Fig. 1 (*a*), (*b*), and (*c*), which shows a stress analysis of a 16-ft. square slab, 6 in. thick. The section, *Y-Y*, is arbitrarily fixed at about 1 ft. less than the breadth of the section. The depth of 6 in. has also been arbitrarily chosen. The section modulus then established defines the moment, using 300 lb. per sq. in. as the allowable fiber stress. Knowing L and the moment, the loads, W and W', are found, which, of course, are much less than the actual safe loads because the subgrade offers substantial resistance to flexure. The diagram, however, is still proportional and representative. The computed sections at x for each foot

Note.—Written discussion on this paper will be closed with the March, 1926, *Proceedings*. When finally closed, the paper, with discussion in full, will be published in *Transactions*.

* With Long-Bell Lumber Co., Longview, Wash.

along L result in the curved figure the area of which outside the square is shaded.

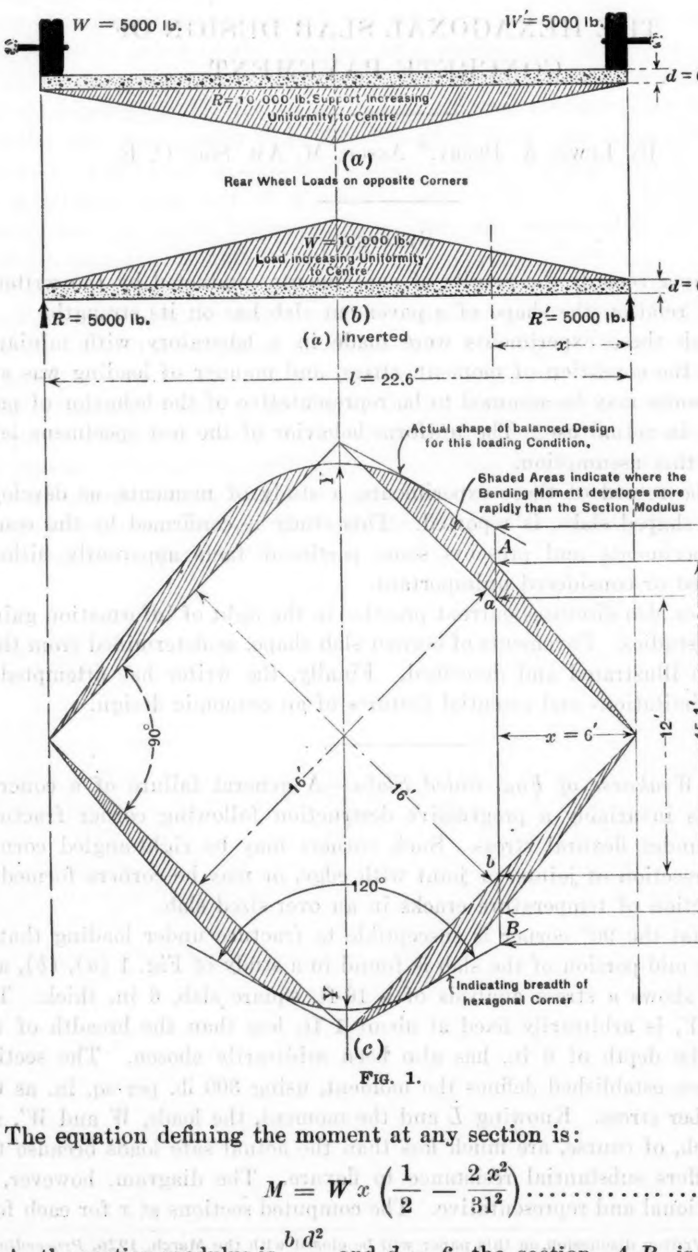

FIG. 1.

The equation defining the moment at any section is:

$$M = W x \left(\frac{1}{2} - \frac{2 x^2}{3 l^2} \right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

As the section modulus is $\frac{b d^2}{6}$, and $d = 6$, the section, $A B$, is defined by

Equation (2):

$$= W x \frac{\left(\frac{1}{2} - \frac{2 x^2}{31^2}\right)}{6 \times 300} = 10\,000 \frac{6\left(\frac{1}{2} - \frac{2 x^2}{31^2}\right)}{6 \times 300}$$

$$= \frac{(60\,000)\,(0.4532)}{1\,800} = 15.1 \text{ ft. } A\,B = \frac{M}{6\,f} \qquad (2)$$

Thus, it is determined that 15.1 ft. of section is needed at AB and the right-angled corner is only 12 ft. at this point, or only 80% efficient.

In his paper entitled "Highway Research in Illinois",* Clifford Older, M. Am. Soc. C. E., emphasized this weakness by stating that corner failures could be prevented by proportioning the thickness of the edge to the thickness at mid-slab as $\sqrt{\frac{3\,W}{S}}$ is to $\sqrt{\frac{W}{S}}$. When W is 8 000 lb. and S is 300 lb., the respective thicknesses should be 8.9 in. and 5.2 in.

Tests of Laboratory Specimens.—In order to determine the relative strength of small specimen slabs, a simple apparatus shown in Fig. 2 was constructed. Square and hexagonal-shaped laboratory specimens, ¾ in. thick, 144 in. in area, and of 1 : 3 sand mortar were made and tested after having been cured 7 days and 5 months, respectively, in open air.

As shown in Fig. 2, the specimens were tested on a bed of dry fine sand, the loads being applied at two points, each 1 in. inside the actual corner, and on a line joining opposite corners. The loads were applied in units of 5 lb. until the specimens were fractured, the time of each test to failure occupying about 10 min.

Table 1 records the results of the tests on specimens that had been aged 7 days, of secondary tests on the fragments of the 7-day specimens, and the loadings necessary to produce primary fractures on the 5-months specimens.

The broken specimens and the relative breaking loads under primary and secondary loading are shown in Figs. 3 and 4. The compound fracture of Hexagon No. 4 (Fig. 3) was caused by splinters from broken specimens that had become hidden in the sand bed. The compound fractures of Hexagons Nos. 6 and 9 (Fig. 4) were caused by the falling of the bar after primary fracture had occurred. It will be noted from Fig. 3 that all square specimens fractured at the corners and that the hexagonal specimens under primary loading (see Table 1) broke into figures of uniform shape and size.

Present Methods of Compensating Corner Weakness.—The practice of using a center joint in highway and street pavements is logical provision against the development of longitudinal cracks. Such joints, or cracks, form right-angled corners at their intersection with transverse joints, or cracks. To preclude fracture of these right-angled corners without the use of shear bars, or other such compensation, the thickness at the edge must be greater than that required at the mid-portion of the slab. This means that a uniform thickness

* *Transactions*, Am. Soc. C. E., Vol. LXXXVII (1924), p. 1180.

is either insufficient at the corners or extravagant at mid-slab. The additional concrete required to deepen the edge along the center joint is an important item of cost.

The shear-bar connection is probably the most widely used provision against corner breakage along the center joint, but it is far from ideal for several reasons. Neither shear bars that are placed at their closest effective spacing, nor the so-called tongue-and-groove type of joint, can entirely compensate corner weakness because diagonal shear of concrete is the limiting factor.

Under adverse conditions, such as unstable subgrades and frost heaving, where the slabs vary seriously in coincidence, spawling of the concrete destroys the bond of the shear bar. The bar is then rendered ineffective, and the slab has suffered a reduction in section at what may be a critical point.

In time, corrosion at the exposed part, and the crystallization caused by vibration, must disqualify the shear bar as an agency of support, and shorten the life of the pavement, which otherwise would be limited only by wear. Corrosion increases the volume of steel about 90 per cent. This increase in volume produces pressure, and a shear bar in a relatively thin pavement may be expected to spawl off a part of the slab, thereby weakening it.

The cost of shear bars, or metal tongue-and-groove joints, although not the governing consideration, is nevertheless important.

Structural Advantages of Hexagonal Slabs.—In view of the foregoing it would seem that an arrangement of plain concrete slabs, the interior corners of which were of sufficient breadth to preclude corner breakage without costly compensation, would be a logical design. This may be accomplished by arranging a plurality of hexagonal or fractional hexagonal slabs, the interior corners of which are substantially 120°, the 90° corners occurring along the outer edge being compensated by additional thickness of concrete. Such an arrangement is illustrated in Figs. 5 and 6.

Fig. 5 shows part of a pavement, of which each slab is a true hexagon, 314 sq. ft. in area and 6 in. in thickness. This pavement is in use continuously for heavy traffic and storage and has shown no evidence of failure. Fig. 6 shows an adaption of hexagonal slabs for street pavement. As shown by Figs. 3 and 4, the corner was the pronounced place of weakness in the four-sided laboratory specimens, which is likewise true of four-sided roadway slabs.

A study of Figs. 7 and 8 will reveal the marked structural advantages of the hexagonal slab. It will be noted that the areas of these two slabs are practically equal, yet the greatest diagonal length of the hexagon is only nine-tenths that of the square. This reduces the bending moments to a gratifying extent. It is also found that the hexagonal corner develops 1.732 times the section width of the square corner, as shown by comparing the line, *a-b*, in Fig. 7, with the line, *A-B*, in Fig. 8. It will be noted that the perimeter of the hexagon is 4 ft. less than that of the square. In localities where extremes of temperature occur and prepared expansion joint material is used in generous thickness, this saving may amount to as much as 3 cents per sq. yd.

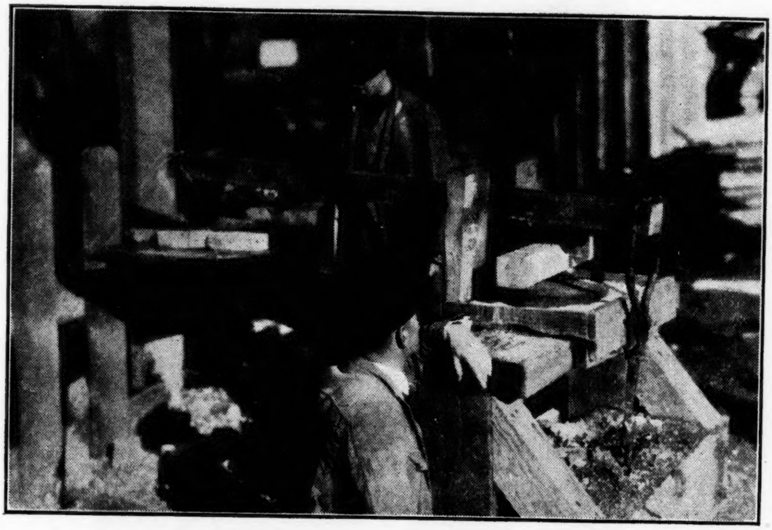

FIG. 2.—APPARATUS FOR TESTING LABORATORY SPECIMENS. NOTE SAND BED.

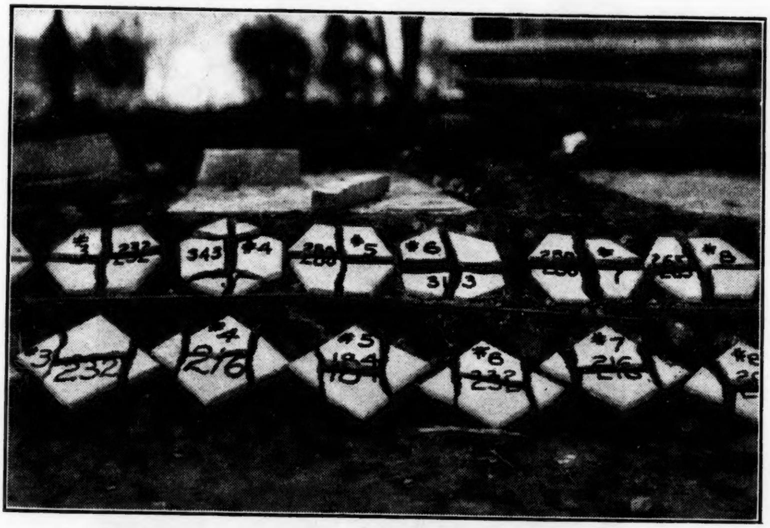

FIG. 3.—GROUP OF TEST SPECIMENS AFTER SECONDARY LOADING. HEXAGON SPECIMENS BROKEN INTO PIECES OF UNIFORM SHAPE AND SIZE.

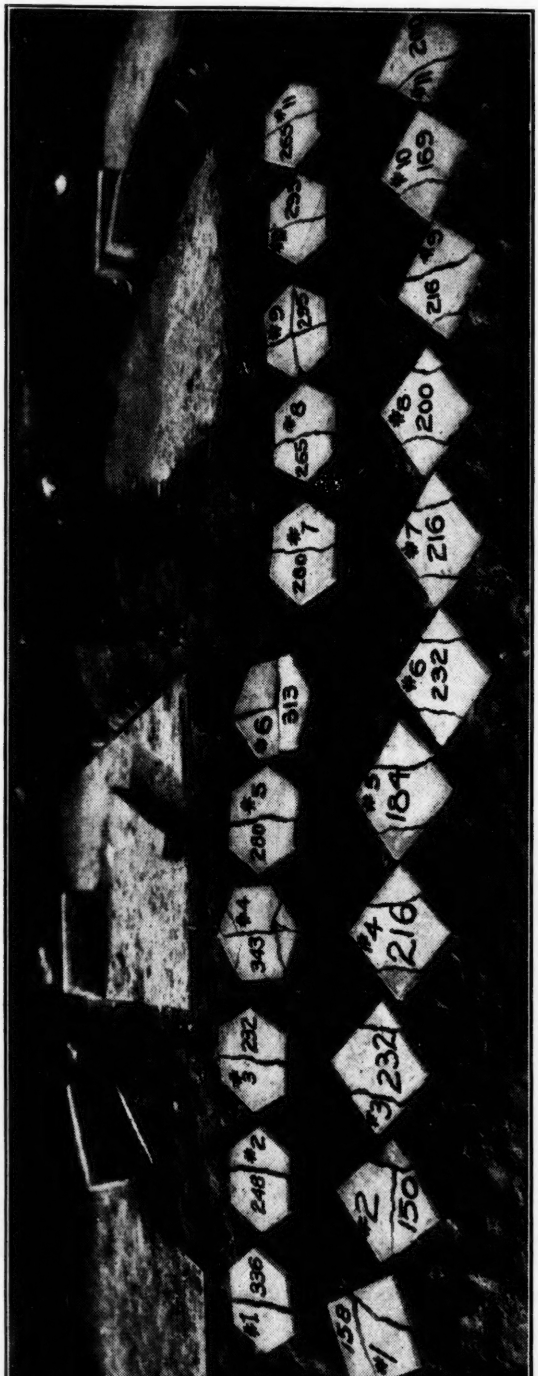

FIG. 4.—VIEW SHOWING ACTUAL BREAKS OF TEST SPECIMENS AND RELATIVE BREAKING LOADS UNDER PRIMARY LOADING. NOTE THAT ALL SQUARE SPECIMENS DEVELOPED CORNER FRACTURES.

FIG. 5.—VIEW OF PAVEMENT OF HEXAGONAL SLAB DESIGN CONNECTING LONG-BELL WEST FIR MILL WITH DOCK "A", UNDER CONTINUAL HEAVY TRAFFIC AND STORAGE.

FIG. 6.—VIEW OF FINISHED STREET BEFORE TRIMMING OF EXPANSION JOINTS.

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TABLE 1.—TEST DATA ON SQUARE AND HEXAGONAL SHAPES OF EQUAL AREA AND THICKNESS—LOADING, IN POUNDS, TO PRODUCE FAILURE.

(Both the square and the hexagonal slabs were constructed of 1 : 3 sand mortar, ¾ in. thick and 144 sq. in. in area.)

	Specimen Numbers.											Total load, in pounds.	Number of specimens.	Average load, in pounds.	Comparative percentage of loads.
	1	2	3	4	5	6	7	8	9	10	11				
Primary Loading, Specimens Seven Days Old.															
Hexagonal slabs.....	672	496	464	687	560	626	560	530	590	590	530	6 305	11	573.1	145
Square slabs.....	316	300	464	432	368	464	432	400	432	438	400	4 346	11	395.0	100
Secondary Loading, Specimens Seven Days Old.															
Hexagonal slabs.....	1 526	1 086	896	.....	1 180	496	898	962	.....	1 086	1 086	9 216	9	1 024.0	187
Square slabs.....	242	590	530	656	338	464	496	464	432	960	866	6 038	11	549.0	100
Primary Loading, Specimens Five Months Old.															
Hexagonal slabs.....	1 330	1 690	1 600	1 660	1 840	1 870	1 660	1 630	1 660	1 480	1 870	18 296	11	1 663.0	167
Square slabs.....	1 090	1 030	670	1 090	1 210	970	940	970	.....	.....	.....	7 970	8	994.0	100

An important virtue of the hexagonal slab pavement is that the greater breadth of corner accumulates the greatest possible area of support near the point of application of the load. In Fig. 9, which is a panel design of a highway or narrow street pavement, the hexagon or fractional hexagon most closely approaches the circle, or semi-circle, of any figure that can be grouped with similar figures. The circle is the most efficient slab shape by reason of the absence of projecting corners. All sections through it are maximum sections. The hexagonal slab, therefore, offers the maximum resistance to fracture under flexure.

Of importance, second only to corner protection, is the virtue of the fractional hexagonal slab, as illustrated in Fig. 9, that the maximum transverse bending occurs on the line, *A-B*. This is true regardless of whether the loading is at *w*, *w′*, or *W*. Assuming the width of the pavement to be 20 ft. and

the staggered joints at the center to be 10 ft., the distance, *A-B*, then becomes 12.5 ft., or 25% greater than the width of a slab with a straight center joint.

Some idea of the greater strength of the hexagonal slab may be obtained from a study of Table 1, and Figs. 3 and 4.

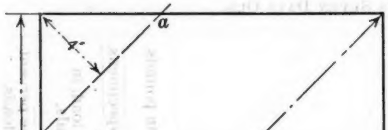


FIG. 7.—HEXAGONAL SLAB.

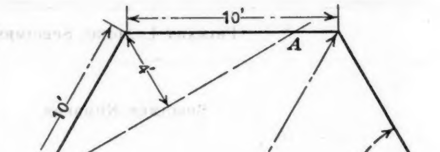
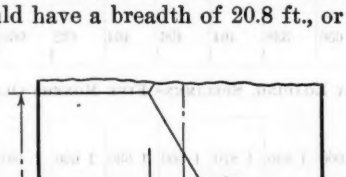


FIG. 8.—HEXAGONAL SLAB.

It will be seen from Fig. 1 (*c*) that although the 90° corner provides only 12 ft. of section along the line, *a-b*, and the actual requirement is 15.1 ft., the hexagonal corner would have a breadth of 20.8 ft., or nearly 38% greater than required.



CROSS SECTION

FIG. 9.—PANEL DESIGN OF HIGHWAY OR NARROW STREET PAVEMENT.

As the section modulus of any member is a correct measure of its strength, the economic possibilities of the hexagonal design are shown by comparing a square slab corner, 7 in. thick, with a hexagonal slab corner, 6 in. thick.

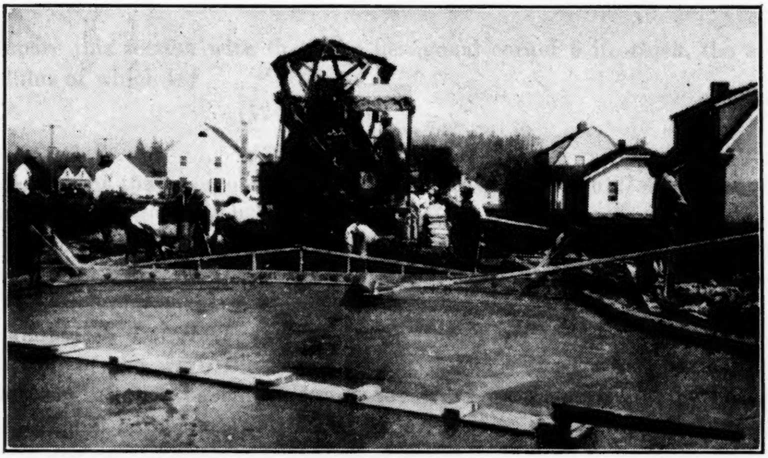


FIG. 10.—HEXAGONAL DESIGN IN STREET INTERSECTION. JOINTS EMBEDDED ½ INCH, ALLOWING UNRESTRICTED USE OF FINISHING TOOLS.

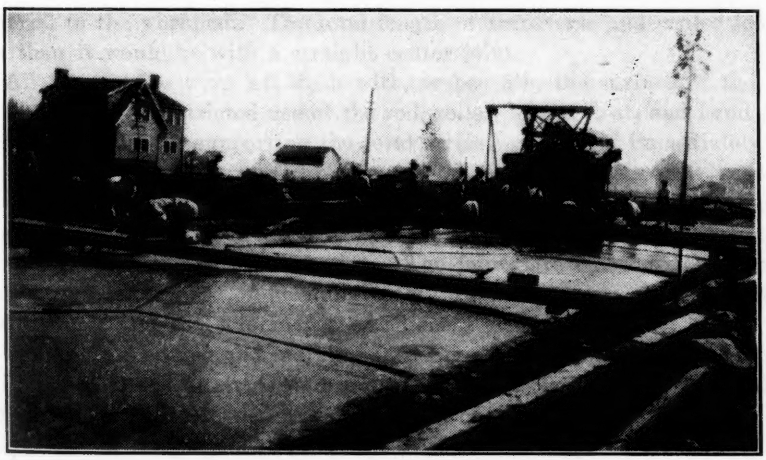


FIG. 11.—EDGING SLABS AFTER JOINT MATERIAL HAS BEEN PULLED.

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Referring to Fig. 1 (c); when x is 3 ft., the section modulus, $\left(\frac{b\,d^2}{6}\right)$, at this section becomes,

$$\frac{(72)\ (49)}{6} = 588 \text{ cu. in.}$$

Compare this section with that of a hexagonal corner 6 in. thick, the section modulus of which is:

$$\frac{(124.56)\ (36)}{6} = 747 \text{ cu. in.}$$

it is seen that the hexagonal slab 6 in. thick has a strength 27% greater at that point than the square slab 7 in. thick.

Another fact deserving mention is that the quality of the concrete itself is likely to be better in the hexagonal corner than in the square corner owing to its better placement in a wider space.

Adaptability of Hexagonal Design to Concrete Pavement.—Before the construction of the work illustrated, the writer discussed with contractors and fellow engineers, a pavement design involving the principle of the hexagonal, or fractional hexagonal, slab. The majority of critics commended the theory of this design, but doubted its practical application with reasonable labor costs. Rarely did one meet with the opinion that the additional cost of constructing such a design could be neglected, yet this is true.

As applied to street and highway pavement, the methods of construction and use of tools used in the laying of the hexagonal slab design are typical of construction of any design incorporating a plurality of units of uniform area. The fact that the center joint is staggered rather than straight is immaterial to the workman. The total length of transverse and center joints is less than it would be with a straight center joint.

All joint strips were left flush with, or beneath, the surface of the pavement to allow unrestricted use of the rod, roller, limber float, and hand floats. The division boards supporting the joint strips were pulled immediately after the rodding. The test straight-edge and all finishing tools were used continuously over the unbroken surface of the concrete, regardless of the location of slab joints, as shown in Fig. 10, which is a view of hexagonal slab pavement being placed at a street intersection. The joints are embedded ½ in., which allows unrestricted use of finishing tools. In this manner the continuity of true contour and the coincidence of adjacent slabs was assured.

Before final edging and marking (see Fig. 11), and while the concrete was still plastic, the joint strips were pulled with flat-billed tongs. This pull was perpendicular and intended to raise the top of the joint strip ¼ to ½ in. above the surface of the pavement.

The construction of the pavement illustrated in Fig. 5 required a somewhat different procedure. A false form system was used for alternate tiers of slabs, the filler tiers being rodded and finished from the existing concrete and with no forms other than division boards. In both these types of improvement it was found that familiarity of workmen with the particular design was not as

important as good organization by the contractor. It was also found that the skilled workman will adapt himself to any logical design and go the designer "one better" in developing practical methods of construction.

The writer has designed and supervised the construction of about 376 000 sq. yd. of hexagonal slab pavement during 1923 and 1924 in Longview, Wash. The question of excessive labor costs due to the irregularity of design is answered by the fact that better progress has been consistently made with this work than with the square panel design constructed during 1923. This comparison was made with equal depths of pavement, and it is worthy of note that the finish and riding quality of the hexagonal slab pavement is, if different, superior to that of the old design.

Credit is due H. O. Root, Principal Assistant, for valuable assistance in design and field supervision of the work illustrated in this paper. Henry G. Niblett, President of Olympic Construction Company, who directed the construction of this work, is deserving of compliment for his co-operation in successfully adapting the design to practice.

Conclusions.—It is incorrect to compensate a pronounced weakness in design, when that weakness may be eliminated.

The right-angled corner is a pronounced weakness and has no rightful place in a concrete pavement, except where it must occur along the outer edge.

The cost to compensate such weak corners successfully is not justified in the light of a more economic design.

Harmful temperature cracks which intersect and form corners in an oversized slab may be avoided by a reasonable reduction of the slab area.

The necessity of forming reasonably small slabs to avoid these cracks may be met without the formation of interior right-angled corners.

The writer believes that the economical design is a grouping of slab units of plain concrete, each unit sized so as to prohibit temperature cracks and excessive bending moments, and entirely independent of, and disconnected with, adjacent slabs, which prevents injury to any one slab by reason of the displacement or failure of its neighbor, and, further, that each slab be cast in the ideal shape—the hexagon.

In this grouping the corner ceases to be a weakness and the general design is logical and economical, with a uniform thickness, except where right-angled corners occur of necessity. This reasoning is presented as the writer's interpretation of simple, natural laws.

STREAM POLLUTION

A SYMPOSIUM*

* Presented at the meeting of Sanitary Engineering Division, Cincinnati, Ohio, April 23, 1925.

A REVIEW OF THE WORK OF THE UNITED STATES PUBLIC HEALTH SERVICE IN INVESTIGATIONS OF STREAM POLLUTION

By W. H. Frost,* M. D.

In March, 1901, Congress provided for the erection of a laboratory by the United States Public Health Service, "for the investigation of infectious and contagious diseases and matters pertaining to the public health," and in the same year a Division of Scientific Research was organized in the Bureau of the Public Health Service. Therefore, the year 1901 may be said to mark the establishment of systematic and continued scientific investigation as a recognized function of the Public Health Service. Considering the rôle which sewage-polluted drinking water was playing at that time in the spread of typhoid fever and other infectious diseases, and recalling that the membership of the Hygienic Laboratory Advisory Board included the great leader in sanitary science, Professor William T. Sedgwick, it was inevitable that attention should have been directed at once to the importance of comprehensive studies of stream pollution in relation to disease. That this was true is evidenced by frequently recurring references in the Annual Reports of the Director of the Hygienic Laboratory during its early years; but the number of other urgent problems was so great and the resources of the Laboratory so limited, that, for several years, work in this field was of necessity limited to occasional studies of local water supplies, undertaken usually in connection with investigations into the causes of the epidemic or endemic prevalence of typhoid fever in various localities.

In 1910, the first systematic investigation of the status and effects of sewage pollution in any large area was begun by the assignment of A. J. McLaughlin, Surgeon, U. S. Public Health Service, to make a survey of cities in the Great Lakes region, with instructions to investigate the extent of the pollution of their water supplies and its relation to the prevalence of typhoid fever and other water-borne diseases, and to examine State and municipal ordinances relating to its control. Upon the completion of these surveys and of the reports thereon, which were published as *Bulletins* of the Hygienic Laboratory, Dr. McLaughlin was assigned, by request of the health authorities of States bordering on the Missouri River, to make a survey of the sewage pollution of that stream. In this work, which was carried out during the summer of 1912, Dr. McLaughlin for the first time had the assistance of another officer of the Service and was enabled, through the co-operation of the health authorities of the States concerned and of certain cities on the river, to establish several laboratories and make a rather extensive series of bacteriological examinations.

* Surgeon, U. S. Public Health Service, in Chg. of Stream Pollution Investigations, Baltimore, Md.

By the time this work had been brought to a close, the International Joint Commission, established under the treaty between the United States and the Dominion of Canada, had taken up the question of regulating the pollution of international boundary waters; and on request of the Commission, Dr. McLaughlin was granted leave of absence from the Service to accept appointment as Chief Sanitary Expert and Director of field work in investigations undertaken by the Commission. These studies, although undertaken independently by the International Joint Commission, may, in a certain sense, be considered as an extension and continuation of the survey of Great Lakes cities previously undertaken by Dr. McLaughlin for the Public Health Service.

In the meantime, by an Act approved August 14, 1912, Congress had extended the function of the Public Health Service to include, among other added duties, that of investigating "the diseases of man and conditions influencing the propagation and spread thereof, including sanitation and sewage and the pollution, either directly or indirectly, of the navigable streams and lakes of the United States"; and in 1913 made a special appropriation, which has since been continued annually, for carrying out these provisions. The Public Health Service was thus enabled for the first time, in 1913, to establish field laboratories at such points in the United States as might be most suitable for special purposes and to employ a scientific personnel especially qualified to conduct investigations in various fields of research.

It was under this extended authority that, in the summer of 1913, a group of sanitary engineers, chemists, biologists, and bacteriologists was assembled and a beginning made on a concerted plan of investigations relative to stream pollution. As originally organized, the work undertaken comprised the following main divisions.

1.—Studies of the bio-chemistry of sewage and industrial wastes were undertaken at the Hygienic Laboratory under the direction of Earle B. Phelps, Affiliate, Am. Soc. C. E., who was appointed in that year as Chief of the Division of Chemistry in the Laboratory. These studies were devoted especially to testing and developing the application of biological oxygen demand determinations to the measurement of the potential polluting effect of sewage and the capacity of streams for its oxidation, a field of research to which Mr. Phelps had already made notable contributions.

2.—Intimately connected with these was a series of studies, likewise under the direction of Mr. Phelps, but carried on for the most part at various points outside Washington, D. C., attempting, by means of experimental installations, to devise better methods for the treatment of various important industrial wastes, for which economical and effective processes had not previously been evolved.

3.—Under the direction of H. S. Cumming, Surgeon, U. S. Public Health Service, the present Surgeon-General of the Service, a study of the pollution and natural purification of the Potomac River was undertaken. The Potomac was selected as a type of tidal stream and special attention was

paid, in this study, to the effect of sewage from the City of Washington on the waters near the mouth of the river where important shellfish beds are situated. This investigation, which was completed in the summer of 1914, was then extended and continued as a survey of the sewage pollution of various coastal waters, with special reference to the contamination of shellfish.

4.—At the same time, in the summer of 1913, work was begun on a study of the pollution and natural purification of the Ohio River, which was selected as a typical large inland stream, receiving sewage, usually without treatment, from all cities on its water-shed, and, at the same time, being used by many of these cities as their source of water supply. Headquarters for this work were established in Cincinnati, Ohio, with subsidiary temporary laboratories at five other points along the river.

These several studies, although conducted by working parties organized into separate units, were closely knitted together by being all under the direction of the Division of Scientific Research in the Bureau of the Public Health Service and by the intimate relations which were maintained between those in charge of the several organizations. In fact, they were considered and pursued, not as separate studies, but as interdependent parts of a common and general plan. They were all continued, substantially as originally organized in 1913, until 1917 when it was necessary to discontinue them in order to utilize their personnel in various other more urgent duties during the period of the World War.

By the latter part of 1919, when it was possible to resume the investigations, the original personnel had become much dispersed by necessary assignments to other duties and by resignations. Likewise, the funds available for these investigations, although not actually reduced to any great extent, were relatively diminished by the material increase in all scales of cost, so that in the re-organization it was necessary to discontinue the investigations of coastal waters, which had been brought to a fairly definite conclusion, and to re-establish the other work at a single base in Cincinnati, which has since served as central headquarters for experimental studies of stream pollution and as the base from which parties have been sent out for work in the field.

Shortly after this re-organization, the Surgeon General, recognizing the need for authoritative advice in the planning and conduct of these investigations, requested Dr. Stephen A. Forbes, Professor Emeritus of Biology at the University of Illinois, and Director of the Illinois State Natural History Survey; Dr. Edwin O. Jordan, Professor of Hygiene and Bacteriology at the University of Illinois; Langdon Pearse, M. Am. Soc. C. E., Sanitary Engineer of the Sanitary District of Chicago; and Earle B. Phelps, Affiliate, Am. Soc. C. E., Consulting Sanitary Engineer, of New York, N. Y., to serve as consultants in studies of stream pollution. These consultants, meeting once or twice each year with the staff engaged in the investigations, and keeping in close touch with the progress made, have rendered generous and valuable assistance in shaping plans, devising methods, and interpreting

results. Subsequently, Joseph W. Ellms, M. Am. Soc. C. E., consented to serve as special consultant in studies of water purification processes and has had an active share in the development of investigations along this line.

Since 1919, the principal field investigations undertaken from this base have been:

1.—A study of the pollution and natural purification of the Illinois River, undertaken chiefly to check and extend observations previously made on the Potomac and the Ohio Rivers relative to the laws governing natural purification in streams.

2.—A survey of representative municipal sewage disposal plants in various parts of the United States, to collect information as to their efficiency and cost in actual operation.

3.—A collective study of municipal water purification plants, chiefly rapid sand filters, as operated in a number of cities on the Ohio River and elsewhere, with a special view to ascertaining more precisely the relations between pollution of the raw water and quality of the effluent under varying processes and conditions of operation.

Along with these field studies, experimental investigations have been consistently pursued in the Cincinnati Laboratory, chiefly along the following lines:

(*a*).—An attempt has been, and is being made, so far without notable success, to reproduce on a small scale, adapted for intensive experimental study, the phenomena of bacterial purification which are now known to take place in natural streams. This has included as a necessary item rather extensive research into the biology of various plankton forms in relation to bacterial purification.

(*b*).—Studies of the biological oxygen demand of sewage, industrial wastes, and polluted river waters have been continued in the endeavor to establish more definitely the laws governing the natural processes of oxidation in streams and to check and improve the precision of methods for making the determinations required.

(*c*).—As an extension of the collective study of municipal filter plants which was completed in 1924, experimental studies are now being made of the relation of the pollution of raw water to the quality of effluent obtainable by rapid sand filtration and chlorination, utilizing an experimental plant on the laboratory grounds which is designed so that the conditions of loading and of operation can be varied at will through a wide range.

In addition to these studies, which have been pursued at Cincinnati, work has been going on for several years at the Hygienic Laboratory, under the direction of Dr. William Mansfield Clark, in a study of the physical chemistry of coagulation, with special reference to applications in water purification.

It would be impossible within a brief space and is, moreover, not pertinent to this paper to relate in more detail the history of the various undertakings which have been outlined, nor will any discussion of the results be attempted. As far as they have matured, they have already been made generally available

in a considerable number of publications,* and some of them with the addition of some more recent data, have been discussed in the papers by Messrs. Theriault, Streeter, and Hoskins, which follow.

In conclusion, it will be more appropriate to review briefly the broad general considerations which have determined the scope and direction of such studies as the Public Health Service has undertaken in this field since it has been in a position to make and pursue any general plan, that is, since 1913.

The first consideration, of course, has been the limitation of available resources, which have sufficed in most years for the maintenance of a staff not exceeding six to twelve workers in the higher grades, enough to form a compact group for consistent work on definite lines, but obviously not sufficient to permit of any wide dispersion. The governing considerations in deciding on the use to be made of these resources have been: The existing status and trend of conditions with respect to sewage pollution in the waterways of this country; the status of sanitary science as applied to devising the remedial measures necessary to meet present and future conditions; and the facilities available through State and municipal organizations, independent institutions for research, and the Engineering Profession at large, for conducting such further investigations as may be required.

With respect to sewage pollution, the status in the United States was, in 1913, and is to-day, that the greater part of the sewage from cities, probably not less than 85 to 90% of it, is discharged without treatment into the most convenient stream. Where the dilution is insufficient for prompt oxidation and removal of the sewage, the result is the establishment of a gross nuisance in the immediate vicinity, offensive to the sense of decency and frequently injurious to the financial interests of the community responsible for the pollution. The remedy for this, however, is at hand, as the ingenuity of sanitary engineers, chemists, and biologists has already devised effective means for the treatment of sewage at reasonable cost, and self-interest may be relied upon to impel cities which suffer nuisance from their own sewage to avail themselves of this remedy. The abatement of such gross nuisance is usually a local matter, requiring no broad plan of concerted action between widely separated communities; and, as the principles of the required treatment are already well established, such special investigation as is required is usually a matter of detail, to ascertain the particular process or combination of processes which will serve most economically and effectively in the particular case. Obviously, such investigations are the business of the State and local authorities and of the practicing engineers retained by them rather than of a Federal agency.

The more usual and more serious result, where dilution and current are sufficient to prevent immediate gross nuisance from the discharge of untreated sewage, is to contaminate the water supplies of other cities taken from the same river system at down-stream points, or, in the case of tidal waters, to dangerously contaminate waters from which shellfish are taken. In

* A list of the more important of these publications is given in the Bibliography appended.

the case of public water supplies necessarily taken from such polluted sources, the immediate remedy is artificial purification of the supply. For this, again, sanitary science has already provided the means in various processes of treatment, economically practicable and of such efficiency that they may be relied upon to give safe effluents from water which is highly but not indefinitely polluted. In 1913 there were, to be sure, a number of cities using dangerously polluted water supplies, but in every instance the remedy—installation of adequate water-purification works—was obvious, and such investigations as were required were not general, to ascertain the practicability of a remedy, but local and special, to decide upon the details of the installation best adapted to apply established principles to the problem at hand. It is clear that these local investigations, like those required for local sewage treatment installations, are not the function of the Public Health Service.

In general, the situation up to the present has been that, notwithstanding the customary practice of discharging raw sewage into streams, those cities which have had to take their water supplies from the rivers thus polluted have almost invariably been able, by applying established processes of artificial water purification, to secure water supplies of good, safe quality. This has been true because the volume of the larger rivers is such as to afford great dilution, even for the sewage of the larger cities, and because the distances between the sewer outlets of these cities and the water-supply intakes of other cities down stream are such as to permit of great reduction in pollution by the natural agencies of purification. Similarly, in coastal waters, although they are grossly polluted in the immediate vicinity of cities discharging sewage, there are still great areas sufficiently free from dangerous contamination to be suitable for shellfish culture. Consequently, local measures, namely, the installation of water-purification plants for safeguarding water supplies and the condemnation or local protection of the relatively small areas unfit for shellfish culture, have sufficed for immediate protection against the dangers of sewage pollution. The protection has not been perfect, but it has tended to become progressively better in recent years, as evidenced by the enormous decrease in prevalence of sewage-borne diseases.

Looking to the future, the conditions foreseen and the remedies which must eventually be applied, become more complex. With the growth of urban population, which still continues at a rapid rate, the sewage pollution of streams and coastal waterways must increase; and sooner or later, in the absence of anticipatory control, it seems inevitable that eventually the pollution will become such that water-purification plants of the highest attainable efficiency will not be able to deliver consistently safe effluents. To guard against this condition it will be necessary, perhaps in the near future, to limit the pollution of such inland streams as are necessary sources of water supply by such measure of sewage treatment as will suffice to keep the pollution at water-works intakes within definite bounds.

This, however, is an extraordinarily complex matter, not only from the administrative point of view, with which this presentation is not concerned, but equally from the scientific viewpoint. It implies a concerted plan of

control applied to an entire river system as a unit, a plan in which, presumably, each community will be required to limit its contribution of sewage pollution, not in the interests of its own citizens, but for the protection of other communities down stream, usually including cities in several States. Safety demands that the measure of control exercised be adequate; justice demands that it be distributed among the communities on some definite and equitable principle, and economy demands that it be not more rigid than is actually necessary to insure the requisite protection to health.

The data needed for laying out any such comprehensive plan for controlling the pollution of an entire river system, with due regard for the considerations of safety, equitable distribution of the burden of control, and economy, are as follows:

First.—It is necessary to have established some quite definite and objective criterion of the quality which is to be maintained in the water supplies taken from the river, as they are delivered to the consumers after artificial purification. This criterion or standard must be in terms of measurable characteristics, determinable by quantitative bacteriological or chemical examinations. It must be rigid enough to insure safety beyond any reasonable question, but not much more rigid than is actually necessary, lest it impose an excessive burden of costs.

Second.—It is necessary to have a fairly precise knowledge of the reliability and efficiency of such purification processes as can be applied at a reasonable cost to purification of the raw water available at the best practicable intake, for it is this efficiency, taken in connection with the standards set for the final effluent, that determines the upper limits of the pollution which may be tolerated at the intake.

Third.—It is necessary to know what proportionate part each of the sewered communities, situated at varying distances up stream, contributes to the pollution existing at any given intake, for otherwise it is impossible to estimate what effect elimination or reduction of the pollution from any single community will have in reducing the pollution in the intake zone. This, in turn, implies a fairly precise quantitative knowledge of the laws governing the processes of natural purification, and of how they may vary in different types of streams in relation to various climatic, seasonal, and hydrographic conditions, for it is only through such knowledge that these great protective processes which Nature has provided may be used most effectively; and not to use them is to waste a natural resource of enormous economic importance.

Unfortunately, sanitary science has not furnished such full and precise knowledge as will be required on any of these points, especially in regard to the natural agencies which tend so greatly and rapidly to reduce bacterial contamination and which constitute one of the main reliances for protection of health. Moreover, it seems unlikely that it will be possible to borrow this knowledge from the experience of other more densely populated countries, as the writer knows of no other country having similar problems in the control of stream pollution on a comparable scale and for a similar

purpose, that will probably have to be studied successfully before a solution becomes necessary for some of the great river systems in the United States.

It is with these considerations in view that the Public Health Service, with the advice of its consultants, has consistently directed its investigations of stream pollution along the lines described, devoting a large part of its effort to such undertakings as the attempt to improve technical methods for laboratory determinations, to evaluate the efficiency of filtration plants under the adverse conditions of loading which may be anticipated in the future, and to add something to the present scanty knowledge of the laws of natural purification. Information of this kind, even if it may seem at this time to be more or less academic, will be essential to sound sanitary engineering practice in the future. Moreover, it appears to be pre-eminently the kind of information that a Federal agency should collect, because it is of general, not local, application, and because it involves such long-continued and laborious investigations as are not likely to be undertaken by private agencies, or even by State and municipal organizations, busy as they are with more immediate administrative work, and with the necessary local studies incident to it.

However, while the Public Health Service is confident that this general policy is sound, it can not, of course, feel equally confident that the sequence which is being followed in the development of these studies is the best possible or that the methods which are being applied are always the most effective. For guidance in such matters the Service relies primarily on its special consultants, but, in addition, it always has sought and sincerely desires the criticism and constructive advice of the entire Sanitary Engineering Profession. Therefore, the opportunity of outlining the purposes and present status of the work to the engineers of the country is especially appreciated, in the hope that they will further it by their criticism and advice.

APPENDIX

BRIEF BIBLIOGRAPHY RELATING TO STUDIES OF STREAM POLLUTION, SEWAGE, AND WATER SUPPLIES.

The following is a list of the publications of the U. S. Public Health Service, relating to studies of stream pollution, sewage, and water supplies. The list includes only publications containing original data, omitting numerous articles which present general discussions of various topics.*

Sewage Pollution of Interstate and International Waters, with Special Reference to the Spread of Typhoid Fever: I, Lake Erie and the Niagara River.
A. J. McLaughlin. (*H. L. B. No.* 77 (1912); 169 pp.)

* The abbreviations used in the Bibliography are as follows: "H. L. B.", *Bulletin,* Hygienic Laboratory, U. S. Public Health Service; "P. H. B.", *Public Health Bulletin,* U. S. Public Health Service; and "P. H. R.," Weekly Public Health Reports, U. S. Public Health Service. The reprint number is given where article has been reprinted separately. All these publications are issued from the Government Printing Office, Washington, D. C.

Sewage Pollution of Interstate and International Waters, etc.: II, Lake Superior and St. Marys River; III, Lake Michigan and the Straits of Mackinac; IV, Lake Huron, St. Clair River, Lake St. Clair, and the Detroit River; V, Lake Ontario and the St. Lawrence River.
A. J. McLaughlin. (*H. L. B. No. 83* (1912) ; 296 pp.)

Sewage Pollution of Interstate and International Waters, etc.: VI, The Missouri River from Sioux City to Its Mouth.
A. J. McLaughlin. (*H. L. B. No. 89* (1913) ; 84 pp.)

Investigation of the Pollution and Sanitary Condition of the Potomac Watershed, with Special Reference to Self-Purification and the Contamination of Shellfish in the Lower Potomac River.
Hugh S. Cumming, with Contributions by W. C. Purdy and Homer C. Ritter. (*H. L. B. No. 104* (1916) ; 231 pp.)

Investigation of the Pollution of Tidal Waters of Maryland and Virginia, with Special Reference to Shellfish-Bearing Areas.
Hugh S. Cumming. (*H. L. B. No. 74* (1916) ; 199 pp.)

Artificial Purification of Oysters.
William F. Wells. (P. H. R., July 14, 1916 ; Reprint No. 351 ; 4 pp.)

Investigation of the Pollution of Certain Tidal Waters of New Jersey, New York, and Delaware.
Hugh S. Cumming. (*P. H. B. No. 86* (1917) ; 147 pp.)

Stream Pollution: A Digest of Judicial Decisions and a Compilation of Legislation on the Subject.
Stanley D. Montgomery and Earle B. Phelps. (*P. H. B. No. 87* (1917) ; 408 pp.)

Treatment and Disposal of Creamery Wastes.
Earle B. Phelps. (P. H. R., December 6, 1918 ; Reprint No. 496 ; 5 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: I, The Treatment and Disposal of Strawboard Waste, by Harry B. Hommon ; II, The Determination of Biochemical Oxygen Demand of Industrial Wastes and Sewage, by Emery J. Theriault and Harry B. Hommon. (*P. H. B. No. 97* (1918) ; 56 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: III, The Purification of Tannery Wastes.
Harry B. Hommon. (*P. H. B. No. 100* (1919) ; 133 pp.)

Studies of Methods for the Treatment and Disposal of Sewage: Treatment of Sewage from Single Houses and Small Communities.
Leslie C. Frank and C. P. Rhynus. (*P. H. B. No. 101* (1919) ; 117 pp.)

A Further Study of the Excess Oxygen Method for the Determination of the Biochemical Oxygen Demand of Sewage and Industrial Wastes.
Emery J. Theriault. (P. H. R., May 7, 1921 ; Reprint No. 594 ; 11 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: IV, The Purification of Creamery Wastes.
Harry B. Hommon. (*P. H. B. No. 109* (1921) ; 87 pp.)

Studies on the Treatment and Disposal of Industrial Wastes: V, The Purification of Tomato-Canning Wastes.
Harry B. Hommon. (*P. H. B. No. 118* (1921) ; 58 pp.)

Hypochlorite Process of Oyster Purification (Experimental).
F. A. Carmelia. (P. H. R., April 22, 1921 ; Reprint No. 652 ; 10 pp.)

The Loading of Filter Plants.
H. W. Streeter. (P. H. R., March 24, 1922 ; Reprint No. 737 ; 13 pp.)

A Study of the Pollution and Natural Purification of the Ohio River: I, The Plankton and Related Organisms.
W. C. Purdy. (*P. H. B. No. 131* (1923) ; 78 pp.)

Sewage Treatment in the United States: Report on the Study of Fifteen Representative Sewage Treatment Plants.
H. H. Wagenhals, E. J. Theriault, and H. B. Hommon. (*P. H. B. No. 132* (1923) ; 260 pp.)

An Experimental Study of the Relation of Hydrogenion Concentrations to the Formation of Floc in Alum Solutions.
Emery J. Theriault and William Mansfield Clark. (P. H. R., February 2, 1923 ; Reprint No. 813 ; 20 pp.)

Indicators for p^H Control of Alum Dosage.
Barnett Cohen. (P. H. R., April 6, 1923 ; Reprint No. 828 ; 2 pp.)

On the Composition of the Precipitate from Partially Alkalinized Alum Solutions.
Lewis B. Miller. (P. H. R., August 31, 1923 ; Reprint No. 862 ; 10 pp.)

A Study of the Pollution and Natural Purification of the Ohio River: II, Report on Surveys and Laboratory Studies.
W. H. Frost, H. W. Streeter, J. K. Hoskins, and R. E. Tarbett. (*P. H. B. No. 143* (1924) ; 343 pp.)

Adsorption of Aluminium Hydrate Considered as a Solid Solution Phenomenon.
Lewis B. Miller. (P. H. R., June 20, 1924 ; Reprint No. 932 ; 14 pp.)

A Study of the Pollution and Natural Purification of the Ohio River: III, Factors Concerned in the Phenomena of Oxidation and Re-aeration.
H. W. Streeter and Earle B. Phelps. (*P. H. B. No. 146* (1925) ; 75 pp.)

The Determination of Dissolved Oxygen by the Winkler Method.
Emery J. Theriault. (*P. H. B. No. 151* (1925) ; 43 pp.)

Some Preliminary Observations from a Study of Water Purification Plants Along the Ohio River.
H. W. Streeter. (P. H. R., January 30, 1925.)

A Study of the Effects of Anions Upon the Properties of "Alum Floc".
Lewis B. Miller. (P. H. R., February 20, 1925.)

THE RATE OF DEOXYGENATION OF POLLUTED WATERS

By Emery J. Theriault,* Esq.

The biochemical oxygen demand test to be discussed in this paper, although at present it enjoys a certain measure of renewed interest, is by no means new. The earliest record of such a procedure is probably to be found in a report published in 1870 by a British Rivers Pollution Commission. In France, oxygen demand determinations were made as long ago as 1885 in a study of the pollution of the Seine. In Germany, extensive series of experiments were conducted on the test from 1900 to 1911. In the United States, a modified procedure appears to have been used in the early experiments at the Lawrence Experiment Station, although it is only since 1915 that the method now in use has been more or less generally adopted.

It is significant both of the intrinsic merit of the biochemical oxygen demand test and, it must be admitted, of the numerous difficulties which arise in its practical application, that, in a recent bibliographical review, no less than 150 references were found which dealt directly with the subject. The consensus of opinion appears to be that the test is valuable. In fact, for the purposes of stream pollution studies, it is frequently the only chemical procedure which can be used to advantage. As a measure of the relative strength of various organic wastes and as a guide in estimating the efficiency of particular methods of treatment, the test also appears to possess decided advantages over the usual chemical procedures.

General Considerations

As regards the theory underlying the test, it is a well established fact that a polluted water containing bacteria, if exposed to air, tends to become completely purified. It has been repeatedly demonstrated that definite quantities of dissolved oxygen are absorbed during this self-purification process. It follows that the quantity of oxygen required for the complete stabilization of a polluted water may be taken as a measure of its organic matter content. In the simplest case, two glass-stoppered bottles are completely filled with the sample under examination. The initial dissolved oxygen content is found by analyzing one of these sub-samples at the beginning of the test. The other sub-sample is placed in a constant temperature chamber at 20° cent. After an arbitrarily selected time, preferably five days, the sample is removed from the incubator and its oxygen content is re-determined. If bacteria and organic matter were present, a decrease in the oxygen content is invariably observed. This decrease is then reported as the 5-day oxygen demand of the sample at 20° cent.

A limitation of this test as outlined lies in the fact that the saturation value for the dissolved oxygen content of water at 20° cent. is only 9 parts

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per million, corresponding to the 5-day oxygen demand of a highly purified effluent or a highly polluted water. With sewage effluents of average quality, a 5-day oxygen demand value of about 20 parts per million may be expected. Before the test can be applied, it is necessary, therefore, to dilute such effluents with five or ten volumes of fully aerated distilled water or tap-water of good quality. For raw sewages, the 5-day oxygen demand is generally greater than 100 parts per million, so that the samples must be diluted about fifty times in order to provide a sufficient supply of oxygen throughout the course of the test. Tannery and abattoir wastes possess oxygen demand values which range from 1 000 to 10 000 parts per million. With unusual trade wastes, 5-day oxygen demand values of 50 000 parts per million have been obtained. At the other extreme, the 5-day oxygen demand of good tap-water is about 0.5 part per million.

Various other methods of procedure have been proposed for determining the oxygen requirements of heavily polluted waters without resorting to dilution. The "excess-oxygen" method just described, inasmuch as it depends on the volumetric determination of dissolved oxygen, using ordinary glass-stoppered bottles, possesses the merit of extreme simplicity. Extensive series of experiments conducted at the Cincinnati Laboratory of the U. S. Public Health Service have amply demonstrated that the precision attainable leaves little to be desired even if it is necessary to dilute the samples before conducting the test. With suitable laboratory facilities, the dilution technique is simple.

A more serious limitation, and a limitation which is inherent in any method of procedure, is the necessity for interpreting the results in the light of time and temperature relationships. Owing to the fact that the rate of absorption of oxygen by a polluted water is exceedingly slow, it is generally desirable to extend the incubation period over several days. Again, as the reaction is purely biochemical, the temperature at which the test is conducted must be carefully controlled. In order to correlate the laboratory results with the ever-changing time of flow and temperature conditions of a stream, it is necessary, therefore, to obtain reasonably accurate formulas by which the oxygen demand of a sample after any interval of time at any specified temperature may be calculated from the values obtained under standardized conditions.

The experiments herein described, were undertaken, primarily, for the purpose of confirming the validity of the various time and temperature correction formulas which have thus far been proposed. The discussion will be limited to the formulas developed in the course of the Ohio River investigation.* These experiments have also demonstrated that factors other than time and temperature must be considered before a valid interpretation of the highly consistent results obtained with the "excess-oxygen" method can be made. In particular, the condition of a sample with respect to its state of oxidation and, possibly, the nature of the micro-organisms present, both exert a marked influence on the magnitude of the observed oxygen demand values.

* H. W. Streeter and E. B. Phelps, U. S. Public Health *Bulletin No. 146*.

Experimental Procedure

For the purpose of securing representative samples, a large vessel was first filled with Ohio River water or, in some instances, with sewage suitably diluted. After the sample had been thoroughly mixed, it was siphoned into bottles with capacities of 350 cu. cm. The initial oxygen content was then determined and the remaining sub-samples were incubated at 9°, 20°, or 30°cent. In the course of experiments which have extended somewhat more than a year, twelve separate series of observations have been made. In most cases, the course of the deoxygenation was followed for at least one month. As a rule the experiments were conducted in duplicate and, in several instances, comparative data were obtained at three different temperatures.

Precision of Base Data

The agreement between duplicate samples was excellent even when the incubation period extended over several months. In one series of experiments in which a large number of sub-samples were titrated after an incubation period of 96 days at 20° cent., the average deviation from the mean was found to be less than 0.2 part per million. The findings in this respect are of considerable analytical interest.

General Course of Deoxygenation Curve

Given the precision of the base data, the next step has been to plot the observed average oxygen demand values against the period of incubation. The type of curve obtained in a typical series of observations is illustrated by Fig. 1. The data plotted in this chart are probably unique in so far as they all refer to the same sample incubated at different temperatures over prolonged periods. It is also to be noted that the oxygen demand determinations were made at relatively short intervals, so that the general course of the deoxygenation curve is reasonably well defined. At 9° cent. (lower curve), there was a slight lag in the establishment of bacterial equilibrium. In other respects, however, there is a striking parallelism between the results obtained at different temperatures.

Considering only the results obtained at 20° cent. (middle curve), it is evident that the rate of deoxygenation decreased very uniformly during the first nine or ten days. Relatively small quantities of oxygen were absorbed during the next five or six days. After the sixteenth day, the rate of deoxygenation suffered a marked acceleration. It is also noteworthy that, contrary to a generally accepted notion, appreciable quantities of dissolved oxygen continued to be absorbed even after the twentieth day. As the same phenomenon has been observed with fully aerated samples, this secondary increase in the rate of deoxygenation can hardly be ascribed to the approaching exhaustion of dissolved oxygen. In fact, within wide limits, the rate of deoxygenation is quite independent of the quantity of dissolved oxygen present.

The evidence accumulated thus far is very favorable to a view emphasized by Adeney and other British experimenters, namely, that under aerobic condi-

tions the stabilization of organic matter proceeds in two distinct and strictly consecutive stages; the carbonaceous matter, etc., is first oxidized; then, and only then, does nitrification set in. The second point of inflection on the deoxygenation curve, therefore, marks the onset of the nitrification stage. It will be convenient to discuss these two distinct stages separately.

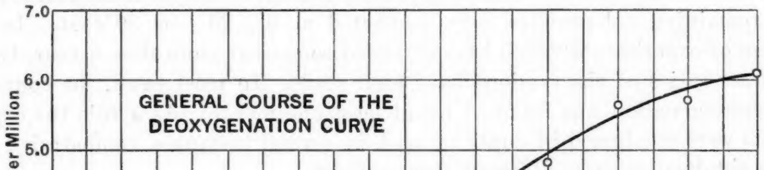

FIG. 1.

RATE OF DEOXYGENATION FORMULA

Considering only the average oxygen demand values corresponding to the first or carbon-oxidation stage, an attempt was next made to determine whether these results conformed with reasonable accuracy to a formula proposed some years ago by Phelps. The formula in question is based on the assumption that the rate of deoxygenation at any instant is directly proportional to the amount of organic matter present in a sample. In the differential notation:

$$\text{Rate of deoxygenation} = \frac{d\,(L_a - L)}{d\,t} = \frac{-d\,L}{d\,t} = K'\,L \ldots\ldots (1)$$

in which,

L_a = oxygen absorbed during the first stage.

L = oxygen requirement of the sample at the time, t.

K = a constant at a given temperature.

The integration of this expression leads directly to the equation:

$$\log \frac{L_a}{L} = \log \frac{L_a}{L_a - X} = K\,t \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which,

X = oxygen absorbed in t days (the value generally reported as the oxygen demand of the sample).

$K = 0.4343\,K'$ = the deoxygenation constant.

Solving for X in Equation (2), the following expression is obtained:

$$X = L_a\,(1 - 10^{-Kt}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

By the aid of tables giving the value of the term, $1 - 10^{-Kt}$, the validity of the Phelps formula may readily be tested. It is only necessary to observe whether a value of L_a exists which satisfies the condition imposed by Equation (3). The agreement between the observed and the computed values is represented graphically by the data plotted in Fig. 2, where the average values obtained in twelve separate series of observations have been recorded. In order to place all values on a comparable basis, and for the sake of avoiding a multiplicity of charts, the results have been plotted, not in parts per million, but as a percentage of the oxygen absorbed during the first stage of the deoxygenation. At each temperature, the line drawn through these average results is simply the graph of the expression:

$$X = L_a\,(1 - 10^{-Kt})$$

For periods of incubation of less than 8 days at 30° cent., 10 days at 20° cent., or 15 days at 9° cent., the agreement between the observed and the computed percentage values is excellent.

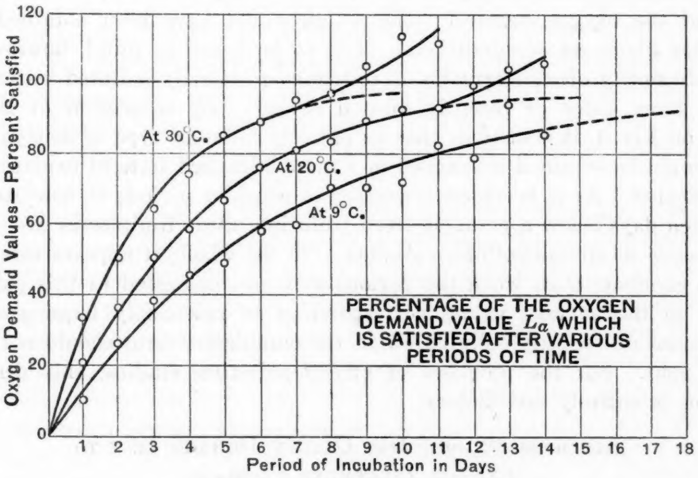

Fig. 2.

Temperature Conversion Formulas

a.—The Value of K at Different Temperatures.—It is also to be noted that in plotting the theoretical curves the value of K was computed by the equation:

$$K_T = K_{20}\,(1.047^{T-20}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

in which,

K_T = the deoxygenation constant at $T°$ cent.

K_{20} = the deoxygenation constant at 20° cent. = 0.100.

The indication is that, in the interval from 9° to 30° cent., the deoxygenation constant is accurately defined in terms of Equation (4).

b.—The Value of L_a at Different Temperatures.—One further point to be considered in connection with Fig. 2 is the value of L_a at different temperatures. Denoting the value of L_a at 20° cent. by 100, the value of L_a at 9° cent. becomes 78 ± 5. Similarly, the relative value of L_a at 30° cent. is 120 ± 7. These values may be represented empirically by the equation:

$$(L_a)_T = (L_a)_{20}\,(0.02\,T + 0.60) \qquad (5)$$

in which,

$(L_a)_T$ = value of L_a at $T°$ cent.

$(L_a)_{20}$ = value of L_a at 20° cent.

The failure to correct for this variation in the oxidizability of a sample with a change in the temperature of incubation does not lead to serious error when the temperature differences are small. In extreme cases a suitable correction can readily be applied.

Applicability of Formulas to Stream-Pollution Problems

Within certain limits, therefore, the possibility exists of converting an oxygen value obtained at any temperature over any period of incubation into terms of the oxygen demand value which would have been obtained under any other given set of conditions. It is to be borne in mind, however, that the applicability of the formulas is restricted to heavily polluted waters, such as raw river water or recently diluted sewage. By inspection of the data plotted on Fig. 1, it is obvious that an entirely different type of deoxygenation curve would be obtained if samples in a more advanced state of oxidation were to be selected. As it is seldom necessary to consider periods of flow exceeding five or ten days below a point of fresh pollution, these limitations are of little consequence in stream-pollution studies. On the whole, it appears safe, therefore, to conclude that, when the various formulas discussed in this paper are applied to the average values corresponding to reasonably large groups of observations on recently polluted water, the cumulative error should not exceed 10 per cent. For the purposes of stream-pollution studies, this degree of precision is entirely satisfactory.

Applicability of 5-Day Oxygen Demand Test to Sewage Treatment Problems

From the foregoing discussion, it may be inferred that for highly polluted waters the oxygen demand values obtained over relatively short periods of incubation possess a clear-cut significance, so that the interpretation of such results offers no difficulty. Attention will now be directed to samples which have reached a higher state of oxidation. The discussion will be conducted with particular reference to sewage treatment problems.

Considering the data plotted in Fig. 1, and assuming that the 5-day oxygen demand of the sample at 20° cent. had been determined only after a preliminary conditioning period of seven days, corresponding to the relatively flat portion of the deoxygenation curve, the observed depletion would have

been about (2.8 — 2.3) = 0.5 part per million. However, if the examination had been delayed for fifteen days, so that nitrification was about ready to start, the observed loss of oxygen would have been about (4.1 — 3.1) = 1.0 part per million. Referred to a sewage effluent which had been diluted fifty times before conducting the test, the two oxygen demand values obtained would have been 25 or 50 parts per million, depending on the amount of preliminary purification which the sample had received. It is noteworthy that under these special conditions the 5-day oxygen demand of the more highly oxidized sample was apparently twice as great as that of the same sample in a less highly purified state. In part, the discrepancy arises from the fact that one set of values has been selected from the relatively flat portion of the deoxygenation curve (8 to 14 days at 20° cent.).

The findings in this respect have a direct bearing on the calculation of the percentage removal of organic matter effected by a treatment plant, and on similar problems in connection with the operation or the comparison of various types of treatment plants. The usual procedure is to base such calculations on the 5-day oxygen demand value of the influent and effluent wastes. In the extreme case in question, it is obvious that the percentage values obtained would stand in inverse relation to the purification actually accomplished. It is not inconceivable that a good measure of the efficiency commonly attributed to Imhoff tanks and similar treatment devices may be due to an effect of this nature. For filter effluents, however, the maximum effect produced by the abrupt change in the slope of the curve may generally be discounted, because the nitrification stage should be fully established when such samples are examined. The possibility of error from this source is nevertheless to be borne in mind.

As regards the time required under laboratory conditions to effect the complete oxidation of the organic matter in a polluted water, definite conclusions can hardly be drawn. On the basis of nitrite, nitrate and free ammonia determinations, it is probably safe to conclude that, at 20° cent., the oxidation of the purely nitrogenous impurity is virtually completed after forty or fifty days. Appreciable quantities of dissolved oxygen, however, continue to be absorbed even after several months of incubation at 20° cent. (See Fig. 3.) The absorption of oxygen beyond the sixtieth day is probably due to the slow oxidation of cellulose-like materials. As it would be impractical to conduct routine tests over such extended periods, it is obviously necessary to conclude that the ultimate oxygen demand of a sample is an indeterminate quantity.

Continuing the discussion of the results derived over long periods of incubation, it appears that when a stage of oxidation has been reached corresponding to that which obtains when a sample of raw sewage is incubated for 30 days at 20° cent., the deoxygenation curve is approximately a straight line. (See Figs. 1 and 3.) The 5-day oxygen demand of a given type of waste, therefore, should be a constant when a sufficiently high degree of purification is reached. It follows that the percentage purification figures computed on the basis of the 5-day oxygen demand test should also tend to be constant when samples in an advanced state of oxidation are examined.

The findings in this respect are in satisfactory accord with the direct observation that the removal of organic matter effected by a representative group of treatment plants was always approximately 90% when partly nitrified effluents only were considered. In view of wide variations in the strength of the raw sewages, in the nature of the treatment devices, and in the methods of operation, this approximate constancy* of the percentage purification values obtained was an unlooked-for result.

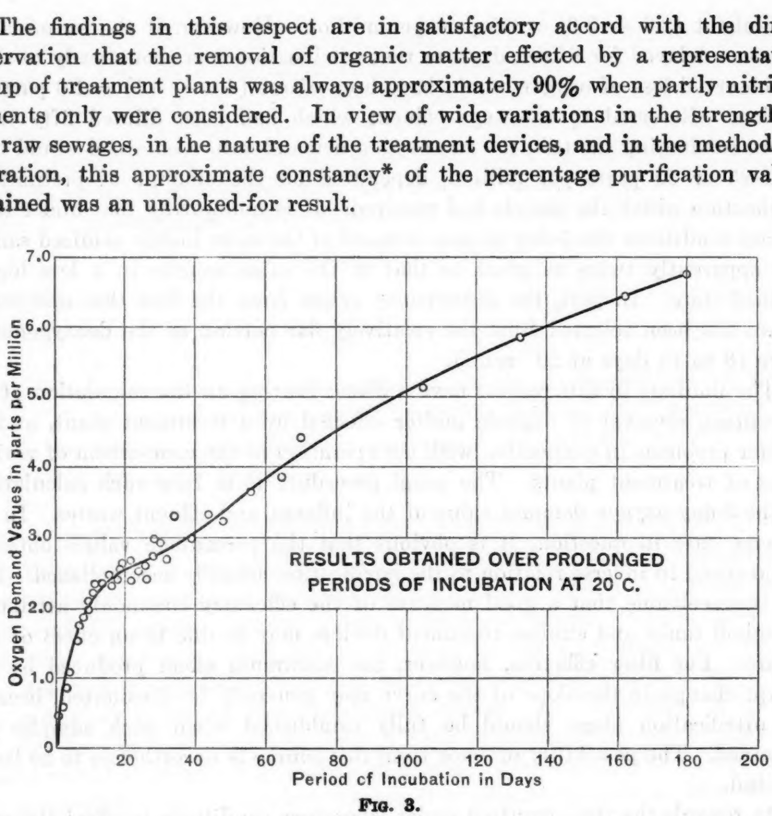

FIG. 3.

Finally, it need hardly be pointed out that a statement to the effect that the 5-day oxygen demand of a sample is, say, 20 parts per million, is of little significance unless a great deal is known concerning the nature or, more precisely, the state of oxidation of the sample. Thus, a 5-day oxygen demand value of 20 parts per million could be referred, with equal reason, to the middle or relatively flat portion of the deoxygenation curve, corresponding to a highly polluted sample, or to the last portion when the nitrification stage has been virtually completed.

Correspondence Between Analytical Data and Observed Natural Conditions

The results thus far presented, although indicative of great uniformity, could hardly be referred to natural conditions without further supporting data. Evidence to the effect that the oxygen demand values obtained during the first stage of the oxidation are directly related to the quantity of organic matter present is given in Table 1. Using the 5-day oxygen demand of a raw sewage as a measure of its organic matter content, and given the contributing popula-

* "Sewage Treatment in the United States", *Public Health Bulletin No. 132*, p. 29.

tion and the total flow of sewage, the per capita contribution of organic matter has been computed for places where fairly accurate data were available. The average per capita oxygen requirement is 51.1 grammes per day, with an average deviation from this figure of 5 grammes. The high value obtained at Columbus, Ohio, is probably due to the presence of relatively large quantities of industrial wastes. Omitting the Columbus result, the average per capita oxygen demand is 48.8 ± 3.1 grammes per day. The constancy of the per capita values is remarkable and leads to the conclusion that the 5-day oxygen demand of a raw waste is directly proportional to the concentration of organic matter present. Moreover, it is apparent that the rate of deoxygenation of diluted raw sewage is not subject to extreme variations; otherwise, the per capita values derived with different sewages would not be consistent.

TABLE 1.—Per Capita Oxygen Demand Values.

(Base data from *Public Health Bulletin No. 132,* p. 115.)

Locality.	Results, in Parts Per Million.			
	5-day oxygen demand actually observed.	Per capita oxygen demand daily.	Deviation from mean, d_1.	Deviation from mean, d_2.
Alliance, Ohio	92	45.6	5.5	3.2
Baltimore, Md	120	45.1	6.0	3.7
Canton, Ohio	213	51.6	0.5	2.8
Columbus, Ohio	190	67.6	16.5	....
Fitchburg, Mass	155	51.6	0.5	2.8
Lexington, Ky	144	48.5	2.6	0.3
Reading, Pa	118	45.1	6.0	3.7
Rochester, N. Y	104	53.9	2.8	5.1
Average*	....	51.1	±5.0	....
Average †	....	48.8	....	±3.1

* To include all observations.
† Omitting the Columbus results.

As regards the general course of the oxidation of organic matter under natural conditions, it is well established that, in sewage treatment, nitrification does not begin until considerable preliminary purification has been effected. Moreover, it has recently been demonstrated in experiments conducted at the New Jersey Agricultural Experiment Station that, even in a filter bed, the onset of the nitrification stage is sharply defined. In the Illinois River investigation, nitrification was not observed until a point far removed from the source of initial pollution had been reached. The exhaustive studies of the Royal Commission on Sewage Disposal of Great Britain also afford instances where the deoxygenation curve represented by Fig. 1 was clearly reproduced in streams. Similar curves were also obtained using undiluted sewage. It appears reasonable to assume, therefore, that the phenomena observed in the laboratory actually correspond to natural conditions.

Conclusions

As a result of the foregoing, the following conclusions have been reached:

1.—The Phelps formula holds with reasonable accuracy when applied to samples recently polluted with organic matter.

2.—For periods of incubation of less than ten days, it is possible to refer the results obtained under standardized laboratory conditions to the actual times of flow and temperatures of a stream.

3.—Under aerobic conditions, the stabilization of organic matter apparently proceeds in two distinct stages.

4.—The rate at which a polluted water is deoxygenated depends largely on the condition of the sample with respect to its state of oxidation.

5.—It is necessary to exercise considerable caution in interpreting the results of analyses when the nitrification stage has almost been reached.

6.—Absolute values for the purification accomplished by a treatment plant cannot be obtained without resorting to protracted incubation.

7.—A complete solution of the problem probably depends on the development of methods whereby the state of oxidation of a sample may be determined more readily.

THE RATE OF ATMOSPHERIC REAERATION OF SEWAGE POLLUTED STREAMS

By H. W. Streeter,* M. Am. Soc. C. E.

Introductory

In all problems of stream sanitation involving the maintenance of an adequate reserve supply of dissolved oxygen for the preservation of fish life or the prevention of nuisance, there are two major factors to be considered as determining the limiting degree of pollution of streams which is consistent with satisfying a given reserve oxygen requirement. One of these factors is the rate of biochemical deoxygenation of the stream water, proceeding in accordance with laws which have been described by Mr. Theriault.† The other element is the rate and extent of replenishment of its oxygen supply from three natural sources:

(a) Dilution water entering the stream through the medium of tributaries and local inflow.

(b) Biological reoxygenation through the activities of certain oxygen-producing plants.

(c) Atmospheric reaeration, or absorption of oxygen directly from the atmosphere.

Of these three sources of oxygen, atmospheric reaeration is by far the most important in freely flowing streams, and this paper is limited to this subject.

It has been widely recognized that atmospheric reaeration is an important factor in the recovery of dissolved oxygen by flowing streams subjected to progressive deoxygenation but, as far as is known, the first effort to evaluate its effects quantitatively as observed under natural conditions, and to correlate such measured effects with the various physical elements which modify them, was made in connection with a survey of the pollution and self-purification of the Ohio River, by the United States Public Health Service, in 1914, 1915, and 1916. The results obtained from this phase of the survey, which recently have been published in the form of a separate report,‡ have served as a basis for a further study of stream reaeration by the Service in connection with a survey of the pollution of the Illinois River, in 1921 and 1922. Although a full analysis of the reaeration data obtained from the Illinois River study has not been completed, it has been carried forward sufficiently to suggest wherein the conclusions reached from the Ohio River study concerning the laws and factors underlying this phenomenon appear to be confirmed and wherein they may require modification. In this paper it is proposed to indicate what both studies have shown, of interest to engineers, as bearing on the theory of stream reaeration and its applications to

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† See p. 1819.

‡ "Studies of the Pollution and Natural Purification of the Ohio River, Pt. III: Factors Concerned in the Phenomena of Oxidation and Reaeration", by H. W. Streeter and E. B. Phelps, *Public Health Bulletin No. 146*, U. S. Public Health Service, Washington, D. C.

problems of river sanitation. For the sake of brevity, the term, "reaeration", will be used hereafter in referring to this phenomenon.

The Nature of Stream Reaeration

The reaeration of flowing streams is governed primarily by the laws controlling the absorption of moderately soluble gases by unsaturated liquids kept in a continuous state of agitation. These laws have been studied recently by a group of chemists, the results of whose observations have been published in the form of a Symposium.* In a paper included in this Symposium, Mr. H. G. Becker† states in the following general form the law of gas absorption which underlies stream reaeration: When a liquid and a moderately soluble gas are allowed to come in contact and the liquid is thoroughly mixed, "the rate of solution of the gas varies directly as the degree of unsaturation of the liquid." In the report on studies of reaeration in the Ohio River, to which reference has been made, it was stated that the rate of solution of oxygen at the surface is directly proportional to the existing saturation deficit (which is merely another way of stating the same law), and it was shown that results obtained by Dibdin and by Adeney and Becker afford experimental confirmation of this principle.

Expressed in terms of stream reaeration, the law thus stated signifies that in each successive unit of time a constant percentage of the remaining deficit in the dissolved oxygen content of the stream below the saturation point, will be satisfied by absorption of oxygen from the atmosphere. The percentage will vary with conditions affecting the rate of absorption, but will remain constant for a given condition. This is analogous to the law of deoxygenation discussed in Mr. Theriault's paper, except that in the latter case the rate of progress of the action is a direct function of the biochemical oxygen demand, rather than the oxygen saturation deficit of the stream water.

In the Ohio River studies, the law of oxygen absorption was formulated thus:

Let,

D_a = the initial oxygen saturation deficit, in terms of concentration;
D = the oxygen deficit at any time, t, expressed in similar terms; and
K_2 = a coefficient defining the rate of reaeration.

Then,

$$\frac{d\,D}{d\,t} = -K_2'' D$$

whence,

$$\log \frac{D}{D_a} = -K_2\,t \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

On referring to Mr. Theriault's paper, it will be noted that this expression is exactly similar to that which defines the rate of deoxygenation, that is,

$$\frac{d\,L}{d\,t} = -K_1\,L$$

* *Journal of Industrial and Engineering Chemistry*, December, 1924, pp. 1215–1230.

† "Mechanism of Absorption of Moderately Soluble Gases in Water", *Journal of Industrial and Engineering Chemistry*, December, 1924, pp. 1220–1224.

whence,

$$\log \frac{L}{L_a} = - K_1 t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

except that, in this case, the biochemical oxygen demand, L, replaces the oxygen deficit, D, and the coefficient of deoxygenation, K_1, replaces the coefficient of reaeration, K_2.

The coefficient of reaeration, K_2, defining the rate of absorption of oxygen, when expressed in terms of oxygen concentration in the stream, has been found, in the Ohio River study, to be modified by stream depth and by various physical conditions which influence the turbulence of flow, among which are the velocity of the current and the slope and irregularity of the channel. In the Ohio River, these relations were found to be governed by a simple equation:

$$K_2 = c\, V^n \times H^{-2} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

in which, V, represents the velocity of flow; H, the depth; and c and n, the constants for a particular river stretch, the values of which depend in part on the channel slope and irregularity. In most cases, it has been found that the value of K_2 is very nearly inversely proportional to the discharge of the stream, which term, multiplied by a proper reducing constant, may be substituted for the square of the depth in Equation (3).

The rate of reaeration is further modified by the water temperature, being accelerated at the higher and diminished at the lower temperatures. The controlling element in this temperature effect appears to lie in the fact that the rate of absorption of oxygen at the surface is limited by the process of diffusion, which, as shown by Black and Phelps,* is governed by a similar temperature relation. It was found in connection with the Ohio River study that when observed values of the reaeration coefficient, K_2, are corrected in accordance with the factors developed by Black and Phelps, the corrected values are more closely correlated with the other stream conditions which have been noted than the uncorrected ones. A few results obtained from the Illinois River study have indicated that the rate of reaeration of this stream does not appear to be influenced as much by seasonal changes in temperature as connections based on the diffusion factors developed by Black and Phelps would imply. However, the results of the recent experiments by Becker, previously mentioned, and by Haslam, Hershey, and Keen,† carefully conducted under physical conditions closely approaching those of flowing streams, have confirmed the earlier findings of Black and Phelps in respect to the direction, and, roughly, to the extent of the temperature effect. As these experimental results are based on far more carefully controlled observations than would be possible under natural conditions, they must be interpreted, for the present at least, as affording a reasonably accurate index of the influence of temperature variations on the rate of reaeration of streams. From a plot of the data compiled by Becker, converted to terms of the reaeration

* W. M. Black and E. B. Phelps, Report on Discharge of Sewage into New York Harbor, to the Board of Estimate and Apportionment, New York City, 1911.

† *Journal of Industrial Engineering Chemistry*, December, 1924, pp. 1224–1230.

coefficient, K_2, the following temperature correction equation has been derived:

$$K_{2\,(T^{\circ}\text{ cent.})} = K_{2\,(20^{\circ}\text{ cent.})} \times [1.0159^{(T-20)}] \ldots\ldots\ldots\ldots (4)$$

This equation is proposed tentatively as probably representing most nearly, from available data, the effect of temperature variations on the value of the reaeration coefficient, K_2, under natural stream conditions. In Fig. 4 is shown a plot of this temperature function as compared with a similar plot of temperature correction factors affecting the rate of deoxygenation, which was developed in connection with the Ohio River studies and has been discussed in Mr. Theriault's paper.

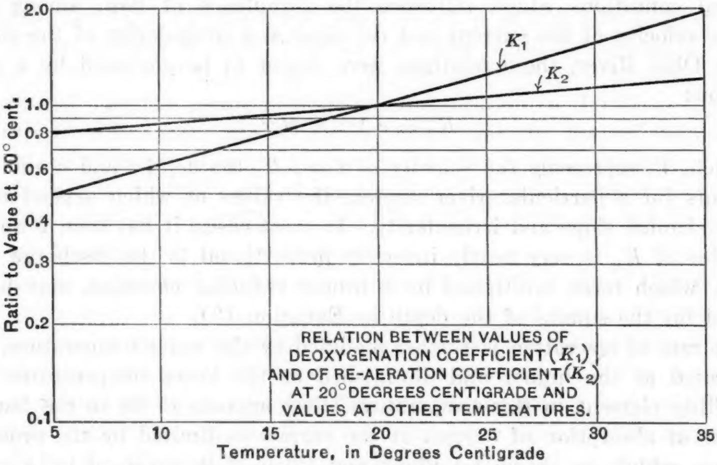


FIG. 4.

EMPIRICAL MEASUREMENT OF THE REAERATION RATE

From what has been stated concerning the extent and modes of action of atmospheric reaeration in streams acting as receivers of community wastes, it is fairly obvious that no even reasonably accurate estimate can be made of the ability of a particular stream to maintain a specified minimum of reserve oxygen supply under a given degree of pollution without a definite knowledge of its capacity for reaeration. This thought leads to a consideration of available means for measuring the reaeration capacities of streams.

Owing to the fact that the rate of reaeration is influenced by a complexity of natural conditions, such as have been noted, methods of laboratory study that have been found suitable for determining the deoxygenation rate are not applicable in this case; hence recourse must be had to measurements in the stream.

If a sufficient number of representative streams could be found in which progressive deoxygenation was not a complicating element, the solution of this problem would be comparatively simple, involving merely the observation of the rate of increase in the dissolved oxygen content of a river between two or more sampling points located at known time intervals of flow from each other. Unfortunately, such a condition never exists, for reasons which are obvious.

The true rate of reaeration, then, is always masked, as far as its observable effect on the dissolved oxygen is concerned, by having superimposed on it a rate of deoxygenation acting simultaneously in the opposite direction.

In order to take account of this condition, an equation was devised during the Ohio River studies, whereby the resultant effect of two given rates, one of deoxygenation and the other of reaeration, on progressive changes in the dissolved oxygen content of a stream can be calculated. This equation was derived by combining the differential expressions, Equations (1) and (2), into a differential equation and integrating it to a variable time, t. The equation thus derived is:

$$D = \frac{K_1 L_a}{K_2 - K_1} (10^{-K_1 t} - 10^{-K_2 t}) + D_a \times 10^{-K_2 t} \qquad (5)$$

in which,

D_a = the initial dissolved oxygen saturation deficit, in terms of concentration;
D = the dissolved oxygen deficit after time, t, in similar terms;
L_a = the initial biochemical oxygen demand;
K_1 = the coefficient of deoxygenation; and
K_2 = the coefficient of reaeration.

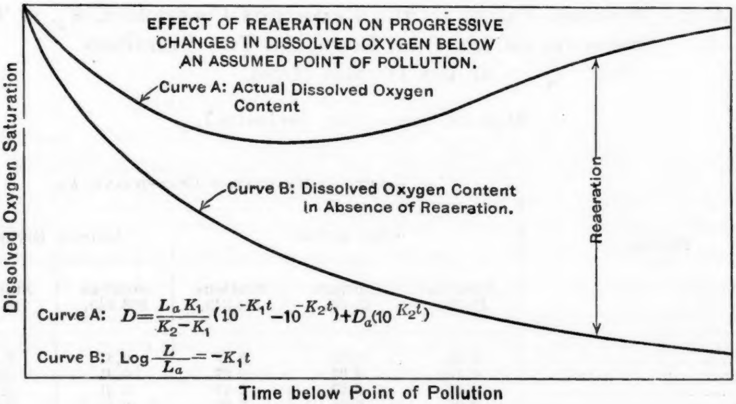


FIG. 5.

The type of curve defined by this equation is shown by Curve A in Fig. 5, which has been reproduced from the report of the Ohio River studies to which reference has been made. For comparison with Curve A, is shown Curve B, representing the progressive deoxygenation which would occur in the absence of reaeration. Curve A is characteristic of progressive changes in the dissolved oxygen content of streams which frequently have been observed in streams below points of major pollution, for example, in the Illinois River below the outlet of the Chicago Drainage Canal; also, in the White River below Indianapolis, Ind. Curve B is characteristic of conditions occasionally occurring in highly polluted streams when covered by a continuous ice sheet, temporarily cutting off reaeration.

By substituting in Equation (5) known or observed values of all terms except that of the reaeration coefficient, K_2, the latter can readily be computed for a given river stretch. A large number of calculations of this kind were made for a series of stretches of the Ohio River, based on observations of the dissolved oxygen and the oxygen demand at the terminals of each river section and on assumed values of the deoxygenation coefficient, K_1, derived from laboratory studies such as have been described by Mr. Theriault, and corrected to the stream temperature by the equation discussed in his paper. A limited number of parallel computations also have been made for a few stretches of the Illinois River. In Table 2 are shown, for comparison, values of the reaeration coefficient derived in this manner from observations in three stretches of the Ohio River and two stretches of the Illinois River presenting, approximately, similar flow and channel characteristics. The results in both cases cover the summer seasonal period, May to September, inclusive. A marked similarity is shown between values of K_2 thus derived in the two streams. It is also noteworthy that the rates of reaeration observed in these five river stretches are approximately double the corresponding rate of deoxygenation as measured by the laboratory value of the coefficient, K_1; thus, the mean value of K_2 is approximately 0.24, whereas that of K_1, at the average river temperature for the given period, is about 0.12.

TABLE 2.—Measured Values of the Reaeration Coefficient, K_2, in Three Stretches of the Ohio River and Two Stretches of the Illinois River.

(May to September, inclusive).

Month.	Values of Reaeration Coefficient, K_2.				
	Ohio River.			Illinois River.	
	Stations 11-19.	Stations 23-65.	Stations 104-349.	Stations 263-240.	Stations 148-122.
May	0.25	0.20	0.18	0.31	0.47
June	0.19	0.33	0.27	0.31	0.28
July	0.29	0.23	0.21	0.21	0.20
August	0.22	0.26	0.21	0.19	0.27
September	0.14	0.19	0.17	0.31	0.14
Mean	0.22	0.24	0.21	0.27	0.27

The locations of river stretches are, as follows:

Ohio River (River miles below confluence of Allegheny and Monongahela Rivers):

Stations 11- 19......Below Pittsburgh, Pa.

" 23- 65......From above mouth of Beaver River to above Steubenville, Ohio.

" 104-349......From below Moundsville, W. Va., to above mouth of Scioto River.

Illinois River (River miles above mouth):

Stations 263-240......From opposite Morris to opposite Ottawa, Ill.

" 148-122......From Pekin to Havana, Ill.

Under some conditions, as, for example, where a stream flows rapidly over a shallow "riffle", the rate of reaeration may become greatly accelerated owing to the diminished depth and increased turbulence of flow. An instance of this kind is found in a short stretch of the Des Plaines River immediately below Joliet, Ill., where the channel is steep and rough and a series of shallow rapids is formed. Calculations of the value of K_2 for this section of the river, based on daily observations extending over a period of ten months, from August, 1921, to April, 1922, inclusive, have given indicated rates of reaeration roughly ten times those observed in deeper and less turbulent stretches of the Illinois River down stream. During the period of December to April, when conditions were most favorable for measuring the true rate of reaeration in this stretch of the river, the following values of K_2 were obtained:

December	2.42
January	2.63
February	2.70
March	2.83
April	2.25
Mean	2.57

The average value of K_2 for the full 10-month period was 2.00.

In general, optimum conditions for determining empirically the value of the reaeration coefficient exist where a stream contains a measurable quantity of dissolved oxygen and where the channel bottom is relatively free from unstable and readily oxidizable sludge deposits. When a stream is wholly or nearly depleted of dissolved oxygen and its channel contains any considerable quantities of decomposing sludge, a very sizeable proportion of the atmospheric oxygen absorbed by such a stream may be withdrawn from solution almost immediately and thereby fail to be accounted in terms either of reserve oxygen or of biochemical oxygen demand. Under such circumstances, the measured value of the reaeration coefficient may be widely in error and always will be lower than the true value. Where an excessively polluted stream contains a measurable supply of oxygen and is relatively free from sludge deposits during a part of the time, measurements of its reaeration capacity should be made when it is in this condition.

Applications

The most important applications of the theory outlined in this paper are found in the estimation of dilution or sewage treatment requirements to be met at specified points along excessively polluted streams to avoid over-taxing their capacities for maintaining a specified reserve oxygen supply, or, conversely, in the calculation of the future limiting permissible degree of pollution

of streams now in a satisfactory condition from this standpoint. Both cases are similar in that they involve the prescription of a limiting biochemical oxygen demand of a stream at certain critical points. As the rate of deoxygenation is accelerated during the summer season to a greater proportionate extent than the rate of reaeration (the latter often is actually retarded during this season owing to a greatly diminished stream flow), conditions during the summer ordinarily are the most critical to be considered in this connection.

In Fig. 6 is given an example showing the effect of temperature variations on progressive changes in the dissolved oxygen as calculated by Equation (5), assuming an initial oxygen demand, L_a, of 20 parts per million and an initial oxygen saturation deficit of zero. The values of the deoxygenation and reaeration coefficients, K_1 and K_2, have been assumed to be 0.10 and 0.20, respectively, at 20° cent. and have been corrected for temperature in accordance with the factors shown in Fig. 4. The time required to attain the maximum oxygen deficit is shown to vary from about 2 days at 30° cent. to 5 days at 5° cent.

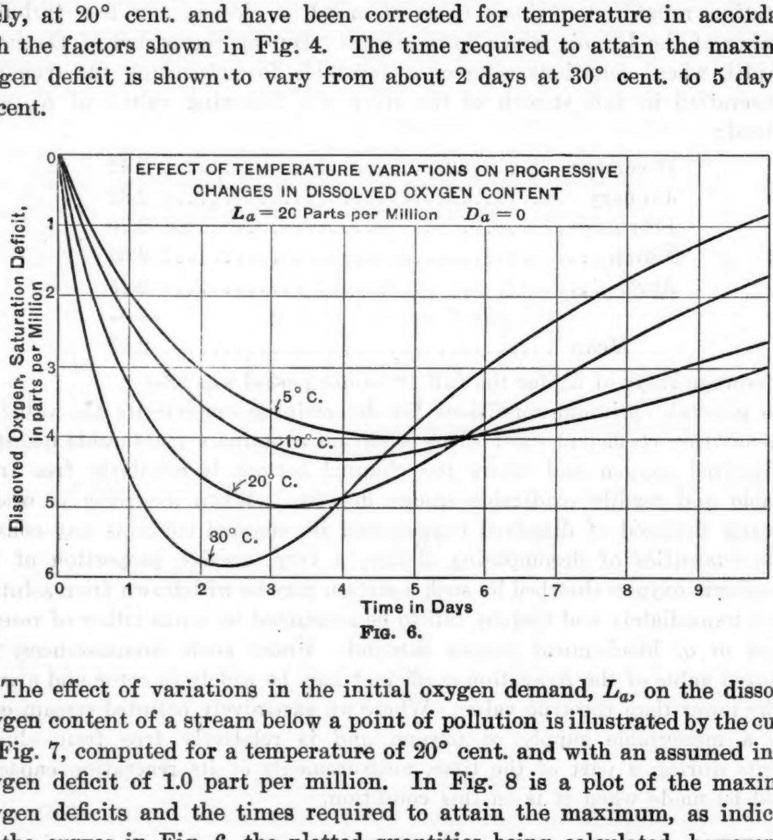
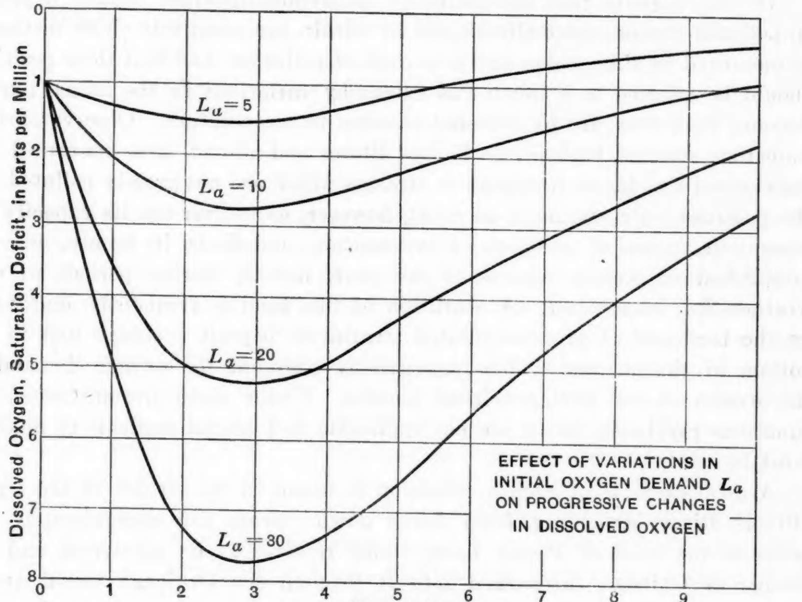


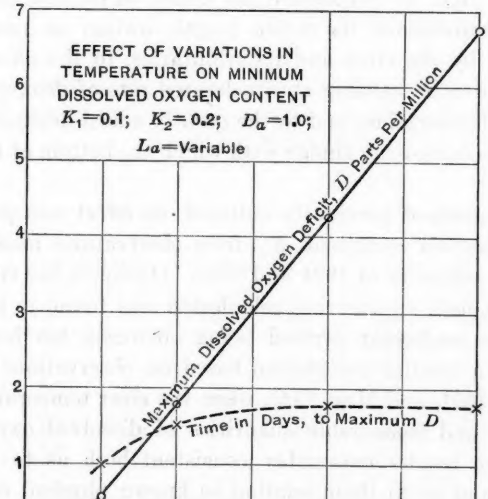
FIG. 6.

The effect of variations in the initial oxygen demand, L_a, on the dissolved oxygen content of a stream below a point of pollution is illustrated by the curves in Fig. 7, computed for a temperature of 20° cent. and with an assumed initial oxygen deficit of 1.0 part per million. In Fig. 8 is a plot of the maximum oxygen deficits and the times required to attain the maximum, as indicated by the curves in Fig. 6, the plotted quantities being calculated, however, by a formula developed by differentiating Equation (5) and placing the resulting expression equal to zero. In this case it is noted that although the maximum deficit varies almost as a straight-line function of the initial oxygen demand, the time to attain the maximum lies within a comparatively narrow range, that is, between 2 and 3 days.



Dissolved Oxygen, Saturation Deficit, in parts per Million
Time in Days
$L_a = 5$
$L_a = 10$
$L_a = 20$
$L_a = 30$
EFFECT OF VARIATIONS IN INITIAL OXYGEN DEMAND L_a ON PROGRESSIVE CHANGES IN DISSOLVED OXYGEN

FIG. 7.



EFFECT OF VARIATIONS IN TEMPERATURE ON MINIMUM DISSOLVED OXYGEN CONTENT
$K_1=0.1$; $K_2=0.2$; $D_a=1.0$;
L_a=Variable
Maximum Dissolved Oxygen Deficit, D Parts Per Million
Time in Days, to Maximum D
Initial Oxygen Demand L_a in Parts per Million

FIG. 8.

It thus appears that the points of maximum dissolved oxygen depletion in polluted streams normally should lie within comparatively short distances, as measured by time, below major sources of pollution, and that their positions should be affected to a much less extent by variations in the initial oxygen demand than they are by seasonal changes in temperature. Observations on numerous streams, both in the United States and abroad, have confirmed this statement in so far as it applies to streams which are not grossly polluted. If the pollution of a stream is so great, however, as to over-tax its capacity for reaeration, zones of complete deoxygenation, indefinite in length, may be established at certain seasons of the year, notably during periods of dry-weather flow in summer. A condition of this kind is frequently aggravated by the tendency of grossly polluted streams to deposit a sludge mat in the bottom of the channel which may greatly augment the oxygen demand of the stream proper during critical seasons. Under these circumstances, the equations previously noted are not applicable and special methods of analysis must be used.

A good example of such a condition is found in the stretch of the Upper Illinois River, extending from Joliet down stream for approximately 110 miles to the head of Peoria Lake, which receives at its up-stream end the sewage of Chicago, discharged into it through the Drainage Canal and a stretch of the Des Plaines River channel. During eight months of the year, October to May, inclusive, this stretch of the river contains a measurable, although in places low, reserve supply of dissolved oxygen. During the four summer months, June to September, its dissolved oxygen content is practically exhausted throughout its entire length, owing, in part, to the lower dilution provided by the river and its tributaries, to the effect of the higher summer temperatures in causing an accelerated rate of deoxygenation as compared with that of reaeration, and to the greatly added deoxygenating effect of the dense mat of decomposing sludge with which the bottom of the river channel is covered.

Following the method previously outlined, an effort was made to calculate values of the reaeration coefficient, K_2, from observations made in the Illinois River during the summers of 1921 and 1922. Owing to the conditions at that time, previously noted, an accurate calculation was found to be impracticable, the values of the coefficient derived being obviously too low and, in some cases, negative. A similar calculation based on observations during the two months, October, 1921, and May, 1922, when the river temperatures approached those of summer and measurable quantities of dissolved oxygen were found in the river, gave results reasonably consistent both as to their agreement with each other and as to their relation to known physical conditions in the several river stretches. From these results the following values of K_2, converted to their equivalents at 20° cent., were derived for the five river stretches forming the upper section of the Illinois River between the limits stated (the station numbers referring to the locations, in stream-miles, above the mouth of the Illinois River):

River stretch.	Value of K_2
Stations 286-263........	0.68 (mean of October and May)
" 263-240........	0.33 (" " " " ")
" 240-227........	0.15 (" " May)
" 227-196........	0.23 (mean of October and May)
" 196-179........	0.14 (" " May)

Although it is likely that the values thus derived (especially the lowest two) are affected to some extent by excessive and unaccountable deoxygenation due to sludge deposits, they are believed to be as nearly representative of the true rates of reaeration prevailing in the several river stretches as any other figures obtained from the present very incomplete series of calculations.

With the foregoing derived values of K_2 as a basis, and using the resultant oxygen Equation (5), a computation has been made of the progressive changes in the dissolved oxygen content of the Upper Illinois River occurring in the stretch extending from Station 286, below Joliet, to Station 179, located 107 miles down stream, during each one of the four months, October, 1921, and May, June, and July, 1922. In making the calculation (details of which are omitted for the sake of brevity), the value of the deoxygenation coefficient assumed was based on the laboratory figure in every instance except that of the river stretch from Station 286 to Station 263, for which the mean of the rates of deoxygenation observed in the stream during the two months, October and May, was used. The values of the reaeration coefficient assumed were the same as those just given, corrected to the river temperature. The calculated dissolved oxygen figures at each station are compared with the corresponding results of observation in Table 3 and illustrated graphically in Fig. 9.

TABLE 3.—COMPARISON OF CALCULATED AND OBSERVED DISSOLVED OXYGEN CONTENTS OF UPPER ILLINOIS RIVER AT SUCCESSIVE SAMPLING STATIONS.

Station.	DISSOLVED OXYGEN SATURATION DEFICIT, IN PARTS PER MILLION.							
	October, 1921.		May, 1922.		June, 1922.		July, 1922.	
	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.	Calculated.	Observed.
263.................	8.4	9.1	8.6	8.0	8.6	8.8	6.2	8.6
240.................	6.4	7.1	7.0	6.3	6.4	7.6	4.8	8.0
227.................	6.5	7.4	7.0	6.4	6.5	8.1	4.9	8.0
196.................	4.3	5.3	4.4	4.3	3.6	5.9	3.0	7.5
179.................	4.9	6.6	3.6	4.6	2.6	6.3	2.7	8.1
	DISSOLVED OXYGEN, PERCENTAGE OF SATURATION.							
263.................	20	14	11	19	3	1	28	2
240.................	39	32	28	35	28	15	44	6
227.................	38	29	28	34	27	9	43	8
196.................	59	50	55	61	60	31	65	12
179.................	63	37	63	52	71	26	69	9

On referring to Fig. 9, it is noted that the calculated and observed figures agree with each other closely for May and reasonably well for October, but they diverge widely for June and July. The divergence probably is due largely to the effect of sludge decomposition in the channel during the summer months, as it represents the excess of dissolved oxygen, unaccounted for in terms of reaeration or normal deoxygenation, which has disappeared from the

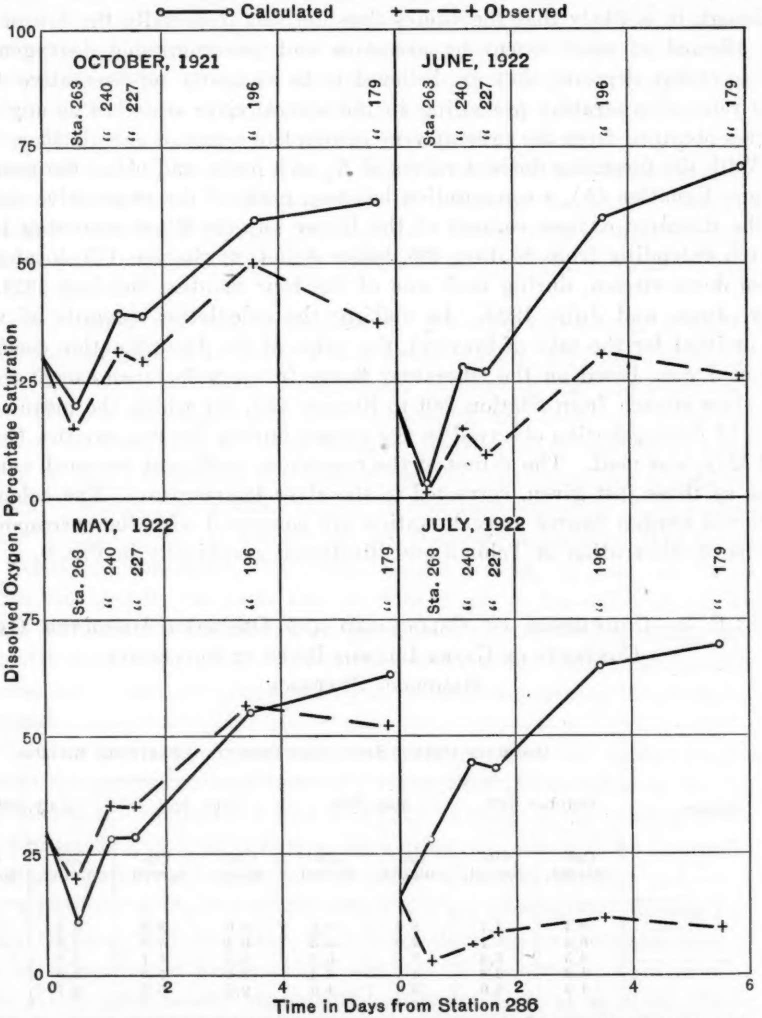


FIG. 9.—COMPARISON OF CALCULATED WITH OBSERVED DISSOLVED OXYGEN CONTENTS AT STATIONS IN UPPER ILLINOIS RIVER. (PLOT OF DATA IN TABLE 3.)

stream in passing from the uppermost to the lowest station and can be accounted for only as oxygen absorbed by the bottom sediments. The deoxygenating power of sludge deposited in the channel is thus indicated as having been sufficient, in July, 1922, to cause an absorption of a quantity of dissolved oxygen equivalent to 60% of the saturation value in a river distance of

107 miles. Although it is hazardous to indulge in speculation in a problem as complex as that presented by the Illinois River, it seems fairly evident that the mere elimination of sludge deposits from the channel of this stream would go far toward restoring the effectiveness of its powers for self-purification.

The density of pollution of the stream proper, however, is fully as important a factor as its condition in respect to sludge deposits in determining its ability to recover its reserve supply of oxygen. To illustrate this point, a series of curves is given in Fig. 10, showing calculated progressive changes in the dissolved oxygen content of the Upper Illinois River with various assumed quantities of initial oxygen demand, the calculation being based on observed conditions at Station 286, below Joliet, during May, 1922. The figures from which Fig. 10 have been plotted are given in Table 4. The comparison is not valid except for purposes of illustration, as any lowering of the initial oxygen demand at Station 286 would necessarily entail improved conditions up stream, which, in turn, would cause an increased oxygen saturation at the point of departure, or *vice versa.* The comparative trends of the curves merely serve to give a rough illustration of the improvement which would be expected if the pollution of a stream at a given point were diminished, without any change occurring in its oxygen status above that point.

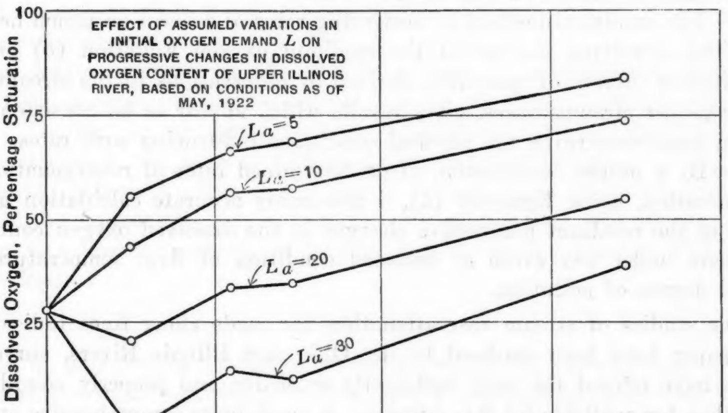


FIG. 10.

In general, it is evident that in almost any given instance where systematic measures are undertaken to relieve excessive stream pollution, a reduction in the oxygen demand of the stream proper and an improvement in its condition with respect to sludge deposits should go hand in hand. This point is an important one to be borne in mind in forecasting the extent of beneficial results to be obtained from extensive stream-cleaning activities. The illustrations given in this paper err considerably on the side of conservatism in this respect, as this fact has not been taken into account in deriving them.

TABLE 4.—Calculated Percentages of Dissolved Oxygen Saturation at Stations in Upper Illinois River, Assuming Different Initial Oxygen Demand Values, L_a, at Uppermost Station.

(Based on conditions as of May, 1922.)

Station.	Time of flow, in days.	Calculated Percentage of Dissolved Oxygen Saturation, with Initial Oxygen Demand, L_a, Assumed as:			
		5 parts per million.	10 parts per million.	20 parts per million.	30 parts per million.
286	0.00	28	28	28	28
263	0.49	56	44	21	1
240	1.08	67	57	35	12
227	1.46	69	58	35	12
196	3.44	84	74	55	39

Conclusions

From the studies briefly described in this paper, the following tentative conclusions appear to be justified:

1.—The reaeration of flowing streams proceeds substantially in accordance with physical laws which have already been described.

2.—Its rate at any time is controlled mainly by the temperature, turbulence, and oxygen saturation deficit of the stream.

3.—The empirical method of measuring rates of reaeration which has been described, involving the use of the resultant oxygen Equation (5) and the substitution therein of quantities derived by observations in the stream made under proper circumstances, gives results which appear to be consistent with known facts concerning the physical conditions influencing such rates.

4.—By a proper combination of predetermined rates of reoxygenation and of reaeration, using Equation (5), a reasonably accurate calculation may be made of the resultant progressive changes in the dissolved oxygen content of a stream under any given or assumed condition of flow, temperature, and initial degree of pollution.

The studies of stream reaeration thus far made along lines indicated in this paper have been confined to the Ohio and Illinois Rivers, surveys of which have offered the only sufficiently extensive and properly co-ordinated data thus far available for this purpose. A much more comprehensive analysis of the Illinois River data, as yet to be completed, probably will give a more satisfactory basis for judgment as to the wider applicability of the results of these studies than it has been practicable to establish within the limited scope of this paper. Some features of the present theory of stream reaeration and its method of application doubtless will require further modification as more experience is gained in testing it against specific problems. The studies thus far completed, however, have indicated that the theory in question, applied with due consideration to its practical limitations, offers a working hypothesis for a much more rational treatment of stream sanitation problems involving the prevention of conditions contributing to nuisance and to the destruction of fish life in streams than hitherto has been available.

QUANTITATIVE STUDIES OF BACTERIAL POLLUTION AND NATURAL PURIFICATION IN THE OHIO AND THE ILLINOIS RIVERS

By J. K. Hoskins,* Esq.

The U. S. Public Health Service has been engaged for some years in studies of various phenomena concerned with the pollution and natural purification of streams. One general purpose of these studies has been to evaluate the intensity of bacterial pollution to be expected from known populations discharging sewage into streams of known discharge and velocity of flow. With this end in view detailed bacteriological data have been collected from two streams of quite different types, the Ohio and Illinois Rivers. Published observations on the Ohio River† covered a period of three years, and those of the Illinois River were continued for a complete year, so that in each instance information was obtained throughout an entire seasonal cycle.

From a consideration of the data of these studies some general tendencies in bacterial changes are indicated, which may be of assistance to sanitary engineers in forming an estimate of the effect, both immediate and prolonged, of adding sewage, from a definite population, to a watercourse of determined hydrometric characteristics.

The degree of bacterial pollution contributed by cities, about which information is most generally desired, may be separated into two principal cases. The first is concerned with the intensity of bacteria that will result in the stream in the zone of highest pollution below the point at which the sewage is discharged. The second and sometimes more important consideration, is the proportion of such contributed bacteria that will remain in the stream at a known distance, or time of flow, below the point at which they were added.

Discussion

Due to fluctuations in discharge and inflow of all streams, the bacterial concentration resulting from a constant rate of contribution may vary widely. It is essential, therefore, for a comparative study of results that not only the concentration of bacteria be considered, but that the actual quantities of organisms be taken into account as well. The quantities of bacteria present in, or added to, a watercourse can be expressed most conveniently in terms of a unit in which is combined the elements of volume, time, and bacterial concentration. Such a unit, called the "quantity unit," has been used for this purpose. The quantity unit may be defined to be the product of the discharge of 1 cu. ft. per sec. and a concentration of 1 000 bacteria

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† "A Study of the Pollution and Natural Purification of the Ohio River, Part II: Report on Surveys and Laboratory Studies", 1924, *Public Health Bulletin No. 143*, U. S. Public Health Service, Washington, D. C.

per cu. cm. Hence, the number of quantity units of bacteria in a stream equals:

$$\frac{\text{Discharge, in second-feet} \times \text{bacteria per cubic centimeter}}{1\,000}$$

Obviously, this unit is convertible into bacterial numbers per unit of time, such as the day. Thus, an average of 1 000 bacteria per cu. cm. in a flow of 1 sec-ft. for 1 day, or 86 400 sec., is equivalent to 28 317 (= number of cubic centimeters in 1 cu. ft.) $\times$ 1 000 $\times$ 86 400, or 2 446 589 000 000 bacteria per day in one quantity unit.

Immediate Pollution

In observations of the effect of pollution by sewered communities, it has been noted consistently that the zone of greatest bacterial density in the receiving stream does not occur immediately below the sewer outfalls, but at a point 10 to 30 hours down stream from the place where such pollution is added. Moreover, the location of this maximum zone seems to be influenced by seasonal temperatures, being farthest down stream during the winter months. Whether an actual multiplication of organisms in the stream takes place until this maximum is reached, or whether the observed increase in bacterial numbers is due to the physical separation of organic matter, has not been definitely determined, although the evidence seems to point to the former assumption as the most logical explanation.

Observations extending over the entire seasonal cycle have been made of the numbers of bacteria per capita added to the stream by the sewage pollution from Cincinnati, Ohio, Louisville, Ky., Chicago, and Peoria, Ill. In each instance the numbers appear to vary with seasonal temperature conditions, being considerably greater in summer than in winter. These seasonal fluctuations are shown for each of the four cities, both in terms of quantity units per capita and in billions of bacteria per capita per day, in Table 5, wherein the values for summer, for winter, and the averages for the entire year, are presented.

By combining the yearly per capita contributions of gelatin, agar, and *B. coli* counts, respectively, of all the four cities, a general average is obtained which may be considered to be roughly representative of the annual average quantity units of the respective types of bacteria contributed to these streams per capita of the sewered population.

The variation from month to month in the numbers of bacteria contributed is, in general, reasonably consistent, increasing quite rapidly to a maximum in June or July and declining again gradually until October and then more rapidly to the lower numbers found throughout the winter season. These changes in the contribution of *B. coli* from each of the four cities are shown in more detail in Table 6, in which the figures represent the ratio of the count each month to the annual average, the latter being taken as equivalent to 100. The averages of the ratios for these four cities for corresponding months represent what might be considered a general measure of the degree of change from month to month in numbers of *B. coli* contributed by urban sewered population. Similar averages have been derived for the monthly

TABLE 5.—SEASONAL CHANGES IN NUMBERS OF BACTERIA ADDED TO STREAMS BY SEWERED POPULATIONS OF CINCINNATI, OHIO, LOUISVILLE, KY., AND CHICAGO AND PEORIA, ILL.

Added by:	Quantity Units of Bacteria per Capita.*			Billions of Bacteria per Capita per Day. Growing on:		
	Gelatin.	Agar.	*B. coli.*	Gelatin at 20° cent. for 48 hours.	Agar at 37° cent. for 24 hours.	*B. coli.*
Chicago:						
Summer	10.148	10.485	0.175	24 828	25 652	428
Winter	1.740	0.346	0.017	4 257	847	42
Year	6.252	4.566	0.094	15 296	11 171	230
Peoria:						
Summer	6.447	10.480	0.0912	15 773	25 640	231
Winter	0.869	3.125	0.0577	2 126	7 646	141
Year	4.518	7.894	0.0763	11 054	19 313	187
Cincinnati:						
Summer	5.764	7.486	0.238	14 102	18 314	583
Winter	1.058	0.410	0.0486	2 588	1 002	119
Year	4.811	5.009	0.1463	11 770	12 256	358
Louisville:						
Summer	5.544	6.475	0 1189	13 564	15 842	291
Winter	3.008	0.3707	0.0789	7 359	907	193
Year	4.431	3.254	0.0907	10 841	7 962	222
Averages:						
Summer	6.976	8.731	0.1565	17 067	21 362	383
Winter	1.669	1.063	0.0507	4 083	2 600	124
Year	5.003	5.181	0.1018	12 240	12 676	249

*One quantity unit = 2 446 589 000 000 000 of bacteria per day.

TABLE 6.—MONTHLY VARIATION IN WATER TEMPERATURE AND IN NUMBERS OF *B. coli* ADDED TO STREAMS BY SEWERED POPULATIONS OF CITIES. (Average for year = 100.)

Month.	Cincinnati, Ohio.		Louisville, Ky.		Chicago, Ill.		Peoria, Ill.		Average.	
	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.	Water temperature, in degrees centigrade.	Bacteria, percentage of annual average.
January	2.4	29	2.6	60	0.3	12	0.6	2	1.5	26
February	3.3	39	3.3	109	1.0	16	0.4	77	2.0	60
March	4.1	32	4.3	91	3.7	13	4.9	228	4.3	91
April	10.9	53	11.3	69	8.6	33	9.6	13	10.1	42
May	17.9	93	18.5	123	15.0	136	18.6	158	17.5	128
June	23.2	116	23.2	79	20.9	200	24.6	494	23.0	222
July	26.2	185	26.8	174	22.4	393	25.8	51	25.3	201
August	25.5	151	26.3	122	23.8	110	25.5	24	25.3	102
September	22.4	111	23.4	174	20.1	123	23.5	64	22.3	118
October	16.7	252	17.5	108	13.8	98	14.5	66	15.6	131
November	9.3	84	11.0	26	7.4	35	7.6	23	8.8	42
December	3.9	56	5.0	66	3.2	30	4.2	0	4.1	38
Year	....	100	....	100	....	100	....	100	....	100

variations in numbers of bacteria growing on gelatin at 20° cent. in 48 hours and an agar at 37° cent. in 24 hours, all of which are assembled in Table 7, and plotted in Fig. 11. For purposes of comparison, the average monthly river water temperatures are also given in Tables 6 and 7.

TABLE 7.—SEASONAL VARIATION IN NUMBERS OF BACTERIA CONTRIBUTED TO STREAMS BY SEWERED POPULATIONS.
(Yearly average = 100.)

Month.	Temperature, in degrees centigrade.	PERCENTAGE OF ANNUAL AVERAGE.		
		Gelatin count.	Agar count.	*B. coli.*
January	1.5	27	13	26
February	2.0	52	20	60
March	4.3	31	22	91
April	10.1	77	32	42
May	17.5	162	107	128
June	23.0	186	227	222
July	25.3	115	168	201
August	25.3	134	188	102
September	22.3	132	161	118
October	15.6	155	166	131
November	8.8	91	69	42
December	4.1	59	27	38
Year	....	100	100	100

If these average values as derived are representative of the bacterial changes in streams in general, as brought about by sewage pollution, they supply a ready means of estimating the maximum concentration of bacteria to be expected in a stream of known discharge resulting from the sewage of a known population. The bacteria per cubic centimeter thus added can be computed at once by the relationship:

$$\text{Bacteria per cubic centimeter added} = \frac{\text{Population} \times \text{quantity units per capita}}{\text{Discharge, in thousands of second-feet}}$$

As an example, the average yearly numbers per cubic centimeter of *B. coli* contributed by Cincinnati (with a sewered population of 494 300) to the Ohio River, where the mean annual flow is 97 500 sec-ft., is:

$$\frac{494\ 300 \times 0.1018}{97.5} = 516$$

Similarly, the concentration for any season or month may be estimated by applying to the yearly average the proper seasonal factor given in Table 7. Thus, in July, when the discharge of the Ohio River is 19 000 sec-ft., the estimated concentration of *B. coli* per cubic centimeter contributed by Cincinnati would be:

$$\frac{494\ 300 \times 0.1018 \times 2.01}{19\ 000} = 5\ 320$$

Applying this method of estimation to the four cities of Cincinnati, Louisville, Chicago, and Peoria, it is possible to compare directly the com-

puted concentrations of bacteria to be expected in each case with the densities actually observed to have resulted from the sewage pollution contributed. Such a comparison of computed and observed concentrations of *B. coli* is presented in Table 8, together with the average monthly rates of stream

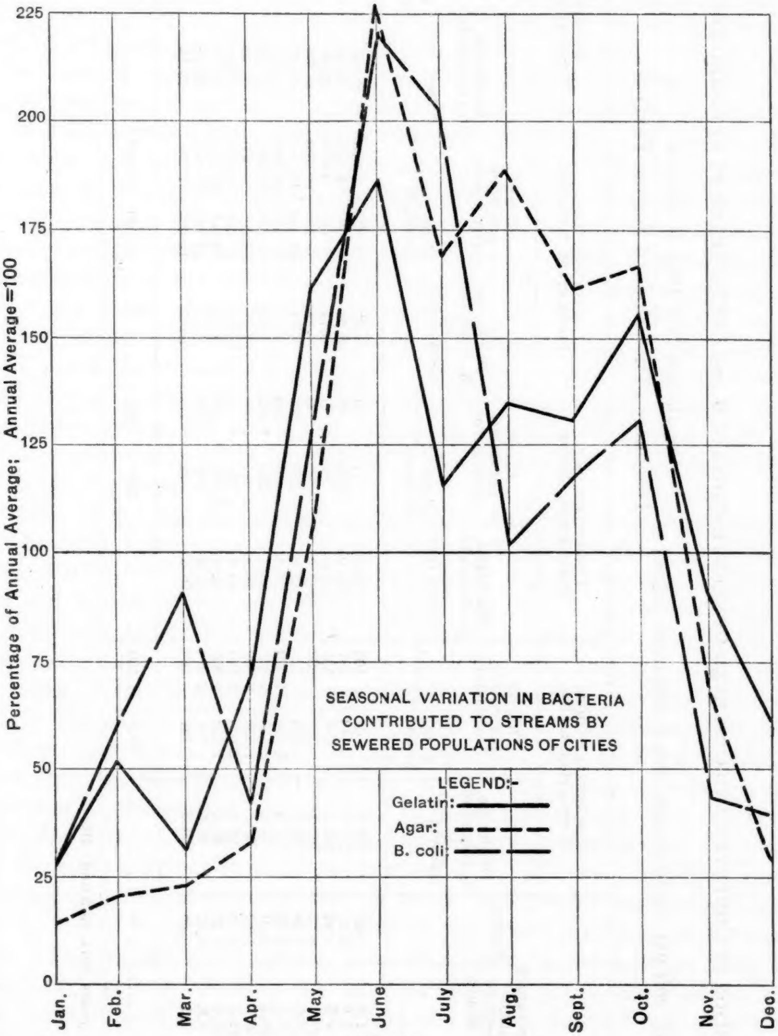

FIG. 11.

discharge and other related data. It will be noted that, although the estimated concentrations for some individual months differ quite widely from those actually observed, yet in the main the two sets of values are comparable and generally of the same degree of magnitude. Bearing in mind that the actual enumeration of *B. coli* is subject to considerable variation, it would appear that the use of the formula as a rough working model may be justified.

TABLE 8.—COMPARISON OF COMPUTED AND OBSERVED CONCENTRATIONS of *B. coli* CONTRIBUTED BY SEWERED POPULATIONS.

$$\left(B.\ coli \text{ per cubic centimeter (computed value)} = \frac{\text{Population} \times 0.1018 \times \text{Factor}}{\text{Discharge (Thousands of Second-Feet)}}. \right)$$

Month.	Temperature range, in degrees centigrade.	Factor.	CINCINNATI,* OHIO, POPULATION, 494 300. Discharge, in thousands of second-feet.	CINCINNATI *B. coli* per cubic-centimeter. Computed.	CINCINNATI *B. coli* per cubic-centimeter. Actual.	LOUISVILLE,† KY., POPULATION, 179 800. Discharge, in thousands of second-feet.	LOUISVILLE *B. coli* per cubic-centimeter. Computed.	LOUISVILLE *B. coli* per cubic-centimeter. Actual.	CHICAGO, ILL., POPULATION, 2 834 000. Discharge, in thousands of second-feet.	CHICAGO *B. coli* per cubic-centimeter. Computed.	CHICAGO *B. coli* per cubic-centimeter. Actual.	PEORIA, ILL., POPULATION, 76 500. Discharge, in thousands of second-feet.	PEORIA *B. coli* per cubic-centimeter. Computed.	PEORIA *B. coli* per cubic-centimeter. Actual.
January............	1.5	0.26	195.8	67	106	108.0	44	91	8.31	9 030	3 890	20.50	99	4
February...........	2.0	0.60	212.7	142	131	221.0	50	80	8.38	20 600	5 200	16.10	290	265
March..............	4.3	0.91	149.7	306	156	192.0	87	77	8.74	30 000	4 050	23.40	303	541
April..............	10.1	0.42	165.5	128	232	297.0	26	38	8.48	14 300	10 400	47.80	68	15
May................	17.5	1.28	92.9	693	720	147.0	159	136	8.91	41 400	40 500	24.40	409	358
June...............	23.0	2.22	68.3	1 640	1 230	28.8	1 410	445	9.37	68 400	56 800	16.90	1 020	1 622
July...............	25.3	2.01	47.1	2 150	2 830	23.1	1 590	1 230	8.85	65 500	118 000	11.70	1 340	241
August.............	25.3	1.02	38.5	1 330	2 880	18.3	1 020	1 090	8.52	34 500	34 500	10.10	786	131
September..........	22.3	1.18	29.4	2 020	2 720	20.9	1 030	1 360	8.23	41 400	39 600	11.90	772	297
October............	15.6	1.31	37.2	1 770	4 900	22.4	1 070	786	8.54	44 300	30 500	12.90	790	284
November...........	8.8	0.42	29.5	716	2 070	12.3	625	340	8.87	13 700	10 400	16.10	203	78
December...........	4.1	0.38	100.3	191	403	96.5	72	111	8.67	12 600	9 200	25.10	118	....
Year...............	....	1.00	....	929	1 530	....	599	482	....	33 000	30 300	....	517	349

* Averages for three years, 1914, 1915 and 1916.

† Data for 1914.

Rates of Decrease in Bacterial Pollution

Quite extensive observations of the decrease of bacteria in polluted waters indicate that such changes follow a fairly regular course, modified by variations in environment, such as temperature and other factors, but yet having an orderly arrangement of reduction. Just what agency is primarily responsible for the death of such bacteria has not been definitely determined. However, there is considerable evidence suggesting that plankton activity rather than lack of food supply is the dominant influence in bacterial diminution.*

A simple and direct method for determining the rates of bacterial decrease in streams, if it were practicable, would be to observe the changes occurring in stored samples of the water under consideration. Unfortunately, the decreases in such stored samples do not correspond invariably with the natural rates occurring in the stream. Long-continued efforts—still in progress—to place the study of bacterial death rates on such an experimental basis, have thus far not been successful. Resort must then be made to the observation of natural purification occurring in streams. Under such conditions, all modifying factors are impossible of accurate control and in many cases corrections for them can be applied only in an approximate manner. Therefore, rates of decrease thus determined must necessarily be interpreted with these limitations clearly understood.

Studies of natural purification of the Ohio River† indicate that changes in the bacterial content between Cincinnati and Louisville are quite orderly and that the rates of decrease can be represented in a general way by empirical curves and formulas. Similar observations on the Illinois River have tended to confirm this conclusion and have indicated that such changes may be of general occurrence, rather than confined to these two streams. The rates of decrease in all instances are not directly comparable, however, and, as stated in the *Public Health Bulletin* referred to, these rates must be considered as approximate only, since they apparently are modified by other factors, such as density or concentration, and perhaps, also, by relative age or staleness of the sewage contributed. It is certain at least that the rates of bacterial decrease from the point of maximum concentration in the Ohio River are quite different from those observed in the Illinois. However, when the disparity in initial concentrations is taken into account and comparisons made at points of equal bacterial density, the rates coincide much more closely. This condition is perhaps best illustrated by the summer rates of decrease in bacteria growing on agar at 37° cent., as observed in the Ohio, in the Illinois River below Chicago, and, again, below Peoria, the base data of which are given in Table 9. Fig. 12 shows these rates plotted from the same origin, and Fig. 13 shows the same curves shifted so that at the points of maximum concentration they coincide with the corresponding density of the Upper Illinois River curve.

Although such an adjustment according to maximum density brings the rates into closer harmony, characteristic preliminary decreases, probably due

* "The Effect of Plankton Animals upon Bacterial Death Rates," by W. C. Purdy and C. T. Butterfield, *American Journal of Public Health*, Vol. VIII, No. 7, July, 1918, pp. 499–505.

† Presented in detail in *Public Health Bulletin No. 143.*

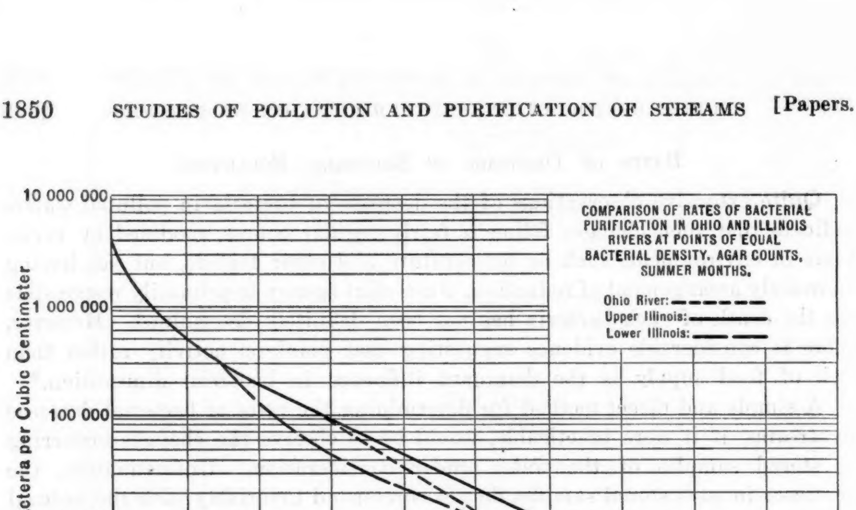
COMPARISON OF RATES OF BACTERIAL PURIFICATION IN OHIO AND ILLINOIS RIVERS AT POINTS OF EQUAL BACTERIAL DENSITY. AGAR COUNTS, SUMMER MONTHS.
Ohio River:
Upper Illinois:
Lower Illinois:
Concentration of Bacteria per Cubic Centimeter
10 000 000
1 000 000
100 000
10 000
1 000
100
0
25
50
75
100
125
150
175
200
225
250
Time of Flow.in Hours

FIG. 12.

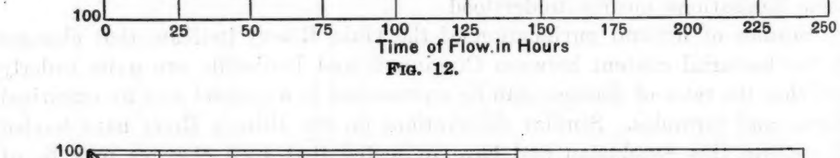
COMPARISON OF RATES OF BACTERIAL PURIFICATION IN RELATION TO TIME OF FLOW FROM ZONE OF MAXIMUM POLLUTION. AGAR COUNTS, SUMMER MONTHS.
Ohio River:
Upper Illinois:
Lower Illinois:
Percentage of Bacteria Remaining
100
10
1.0
0.1
0.01
0
25
50
75
100
125
150
175
200
225
250
Time, in Hours, from Point of Maximum Concentration

FIG. 13.

to other unknown factors, are still evident in each of the curves. It may finally prove to be impracticable, therefore, to develop a composite expression or curve defining accurately the general rate of bacterial decrease in all streams. A series of such expressions or curves, taking into consideration various modifying factors, may be found to best portray the actual rates of most probable change.

TABLE 9.—DECREASE IN AGAR COUNTS, SUMMER SEASON.
(As read from curves.)

Time from maximum, in hours.	Ohio River.		Upper Illinois River.		Lower Illinois River.	
	Per cubic centimeter.	Percentage of maximum.	Per cubic centimeter.	Percentage of maximum.	Per cubic centimeter.	Percentage of maximum.
0	99 300	100.00	3 890 000	100.0	248 000	100.0
10	66 800	67.26	1 180 000	30.3	121 000	48.7
20	45 100	45.37	640 000	16.45	75 000	30.2
30	30 500	30.71	410 000	10.55	52 000	21.0
40	20 800	20.90	275 000	7.07	38 000	15.3
50	14 200	14.31	197 000	5.06	28 200	11.4
70	6 840	6.89	106 000	2.72	16 500	6.65
100	2 540	2.56	43 000	1.10	7 800	3.14
125	1 290	1.30	20 500	0.526	4 200	1.69
150	755	0.76	9 600	0.247	2 150	0.87
175	497	0.50	4 500	0.116	....	......
200	357	0.36	2 150	0.055	....	......

However, the decreases in bacterial numbers are in a broad way quite similar, and it is possible, from the available data collected, to indicate the general trend of such decrease. Such a general rate of purification, if applicable to a specific case, may assist in forming an estimate of the relative numbers of bacteria that may be expected to survive in the stream after any definite interval of time.

TABLE 10.—DECREASES IN *B. coli* FROM VARIOUS MAXIMUM CONCENTRATIONS, IN RELATION TO TIME OF FLOW, SUMMER SEASON.

Ohio River Curve.*		Upper Illinois River.		Lower Illinois River.	
Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.
0	2 280	0	65 600	0	3 550
10	1 459	10	22 200	10	1 300
20	934	20	11 800	20	620
30	599	30	6 700	30	340
40	385	40	4 100	40	190
70	105	70	1 100	70	40
100	32	100	385	100	11
125	14	125	170	125	4
150	8	150	76	......	......
175	5	175	34	......	......
200	4	200	15	......	......

*Table No. 125, *Public Health Bulletin* No. 143.

Such general rates of decrease for *B. coli*, under both summer and winter seasonal conditions, have been outlined by the data of the Ohio and Illinois

TABLE 11.—DECREASES IN *B. coli* FROM VARIOUS MAXIMUM CONCENTRATIONS, IN RELATION TO TIME OF FLOW, SUMMER SEASON.

OHIO RIVER OBSERVATIONS GROUPED BY INITIAL CONCENTRATIONS.*															
Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.
0	8 237	0	4 623	0	2 557	0	1 684	0	1 211	0	607	0	239	0	165
.....	.....	.....	.....	4.7	1 746	.....	.....	4.6	1 091	2.3	325	3.8	138	2.5	77
.....	.....	.....	.....	7.3	1 560	7.3	961	8.9	728	6.0	286	6.3	100	.....	.....
.....	.....	14.9	1 741	.....	.....	11.5	1 264	.....	.....	.....	.....	12.5	70	.....	.....
17.7	1 460	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	18.2	70
24.9	1 548	22.8	1 953	.....	.....	.....	.....	23.5	336	23.6	417	.....	.....	.....	.....
28.6	1 359	29.2	951	.....	.....	25.9	197	.....	.....	27.3	174	28.3	25	.....	.....
.....	.....	36.4	604	.....	.....	38.2	178	38.7	396	.....	.....	.....	.....	35.4	51
.....	.....	43.5	450	49.4	325	45.1	402	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	62.8	429	64.2	88	35.1	73	30.8	98	.....	.....	.....	.....
.....	.....	96	122	.....	.....	.....	.....	84.5	70	.....	.....	.....	.....	.....	.....
110	94	.....	.....	118	21	101	35	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	131	328	136	18	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	.....	.....	159	9	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....
.....	.....	255	5	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....	.....

* Data from Table No. 114, *Public Health Bulletin No. 143*.

River studies, and are assembled in Tables 10, 11, and 12. The rates of decrease for the Illinois River were derived from observations on the Upper Illinois River in which the sewage of Chicago is the agency of pollution, whereas in the Lower Illinois River, the major pollution is contributed by the Metropolitan District of Peoria. Daily observations at successive down-stream sampling stations were averaged over both the summer and winter seasons, and smooth curves defining the rates of natural purification were drawn through these experimentally determined results, plotted on semi-logarithmic paper. The method of obtaining the Ohio River rates of decrease is described in the *Public Health Bulletin* referred to previously. In addition to these general curves of the Ohio, data defining rates of purification at different maximum densities of bacterial content are presented in Table No. 114 of that publication, wherein are presented observations of bacterial numbers at successive sampling stations grouped according to volume of discharge of the river. By grouping these data according to initial concentration and by times from origin, average densities for each group have been obtained at various average intervals of time in hours from the point of maximum density. The results of such grouping for different initial densities are also presented in Tables 10, 11, and 12 for summer and winter, respectively, and define average rates of decrease of *B. coli* starting from the various maximum concentrations.

TABLE 12.—DECREASES IN *B. coli* FROM VARIOUS MAXIMUM CONCENTRATIONS, IN RELATION TO TIME OF FLOW, WINTER SEASON.

OHIO RIVER CURVE.*		UPPER ILLINOIS CURVE.		LOWER ILLINOIS CURVE.		OHIO RIVER OBSERVATIONS GROUPED BY INITIAL CONCENTRATIONS.†					
Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.	Time from maximum, in hours.	*B. coli* per cubic centimeter.
0	260	0	7 200	0	180	0	368	0	231	0	173
...	...	...	.....	...	...	4.6	62	3.4	115	2.3	92
...	...	...	.....	...	...	7.4	133	6.6	98	6.0	86
10	156	10	4 220	10	111	12.8	175	....	...	12.5	130
...	...	...	.....	...	...	....	...	....	...	18.8	66
...	...	...	.....	...	...	....	...	20.5	117	21.6	70
25	78	25	2 500	25	68	....	...	26 1	40	27.3	70
...	...	...	.....	...	...	38.3	46	33.9	86	....	...
...	...	...	.....	...	...	44.3	54	....	...	49.7	41
50	31	50	1 400	50	37	59.3	25	64.2	33	51.6	35
75	17	75	890	75	22	83.3	18	....	...	....	...
100	12	100	610	100	14	104	8	....	...	....	...
125	9	125	422	...	...	131	11	....	...	....	...
...	...	150	300	...	...	...	...	....	...	....	...
...	...	175	210	...	...	...	...	....	...	....	...
...	...	200	150	...	...	...	...	....	...	....	...

* Table No. 127, *Public Health Bulletin No. 143.*

† Data from Table No. 114, *Public Health Bulletin No. 143.*

If these data are plotted on semi-logarithmic paper, in which the concentrations of *B. coli* are plotted as ordinates on the logarithmic scale and the times

from maximum concentration as abscissas on the plain scale, it will be observed that these points define, in a general way, fairly smooth curves, all of which have the same general trend. It may be noted also that the curves of winter purification are of much flatter slope than are those of summer, indicating that natural purification proceeds at much slower rates during cold weather. It is entirely possible, therefore, regardless of the generally lower numbers of bacteria contributed during the winter season, that the critical or most severe period of bacterial pollution to be expected from a given sewage discharged up stream may occur during the winter, rather than during the summer. This is, of course, the reverse of what would be expected of the critical oxygen depletion condition resulting from the same sewage pollution.

Having obtained the general rates of decrease in *B. coli* for various initial concentrations, it is possible to interpolate intermediate rates of decrease, starting from definite initial concentrations. Such interpolated rates should indicate, in a general way, the numbers of *B. coli* that may be expected to survive after definite intervals of time have elapsed beyond the point of greatest bacterial density in the stream. Such rates of decrease are for convenience placed in tabular form for ready reference in Tables 13 and 14, Table 13 representing summer conditions and defining numbers of bacteria remaining after definite intervals of time, starting from various densities at the maximum point, and Table 14 presenting the same data for winter months.

TABLE 13.—NUMBERS OF *B. coli* PER CUBIC CENTIMETER REMAINING AFTER STATED TIMES OF FLOW FROM POINT OF MAXIMUM CONCENTRATION, SUMMER SEASON.

Initial maximum concentration of *B. coli*.	*B. coli* PER CUBIC CENTIMETER REMAINING AFTER INTERVAL OF:								
	10 hours.	25 hours.	50 hours.	75 hours.	100 hours.	125 hours.	150 hours.	175 hours.	200 hours.
75 000	30 000	9 800	2 900	1 000	420	190	84	37	17
60 000	20 000	7 900	2 300	800	350	150	70	32	15
40 000	14 000	5 900	2 000	600	270	120	57	27	14
20 000	7 600	3 000	1 070	420	190	94	47	25	13
10 000	4 000	1 600	640	270	130	68	36	20	12
5 000	2 300	1 100	410	170	78	42	24	15	12
1 000	440	210	80	30	13	5	....	....	....
500	260	130	70	25	8	....	....	....	....
100	40	20	12	....	....	....	....	....	....

However, before such estimates can be accepted with complete confidence, it is obviously necessary that they be checked by observations on a considerable number of streams of different physical characteristics. The empirical results herein presented outline what the observations thus far have indicated to take

place, and endeavor to suggest their practical application. The explanation of the phenomena concerned in such changes must await additional research.

TABLE 14.—NUMBERS OF *B. coli* PER CUBIC CENTIMETER REMAINING AFTER STATED TIMES OF FLOW FROM POINT OF MAXIMUM CONCENTRATION, WINTER SEASON.

Initial maximum concentration of *B. coli*.	*B. coli* PER CUBIC CENTIMETER REMAINING AFTER INTERVAL OF:								
	10 hours.	25 hours.	50 hours.	75 hours.	100 hours.	125 hours.	150 hours.	175 hours.	200 hours.
10 000	6 000	3 500	2 000	1 200	840	600	420	300	200
5 000	3 000	1 800	960	600	400	300	200	140	100
1 000	520	280	140	80	54	38	26	....	....
500	240	120	60	32	21	15	....	....	....
100	62	40	20	12	7	....	....	....	....

SUMMARY

Quite extended observations of the pollution of Illinois and Ohio Rivers have indicated that the numbers of bacteria contributed per capita by the sewered populations of various cities are reasonably constant; these numbers change, however, with seasonal temperature, being much greater in summer than in winter. Such bacteria tend to increase in numbers in the receiving stream for a short period and then decrease at orderly rates as the time from the point of maximum density is increased. These rates of decrease were found to be affected by water temperature and apparently by concentration, being most intensive during the warmer months and under conditions where the density of bacteria was greatest.

Having established definite quantitative relationships from these observations, and assuming that they are fairly representative of stream conditions in general, a method is suggested for estimating the maximum concentration of *B. coli* in streams of known volume of flow that may be expected to result from pollution contributed by known sewered populations. Furthermore, the concentration of such organisms remaining at any point down stream may be estimated, providing the velocity of flow is ascertained.

If the observations are representative of general biological laws, they are of practical value for estimating the increasing burden placed on streams receiving the sewage of growing communities and, consequently, the added loads that water-purification plants must be prepared to handle where such polluted watercourses are used as sources of water supply.

MUNICIPAL WATER SUPPLY PROBLEMS OF ATLANTA, GEORGIA

Discussion*

By Messrs. J. N. Chester, and Nisbet Wingfield

J. N. Chester,† M. Am. Soc. C. E.—The definition of engineering has been stated as "the ability to adapt the forces of Nature to the uses of man", and to that might be added the provisions as well as the forces of Nature. As indicated in the paper, Atlanta has been especially favored with a remarkable source of water supply. The water is slightly contaminated, red with generous proportions of aluminum alkalinity, and the minimum run-off to date has been sufficient for the city, but, at times, the shortage has been acute. This, however, is not alarming considering that there are bountiful storage possibilities above and near the intake, and that in these storage possibilities there also appear to be power possibilities which may some day be developed to avoid the burning of coal for pumping.

With regard to early filtration plants, the Hyatt plant represents more nearly the early stage of the art than any the speaker has seen. There are a great many of these older plants still in operation, but most of them have been shorn of some of the elements of the early development and are operating much as the horizontal filters of the Atlanta plant; but the Hyatt plant shows how filtration was first conducted. At least one unit of that plant should be preserved so that engineers may be able to hearken back to where they started and demonstrate, in a measure at least, that the original filtration plants can to-day produce as good an effluent as the modern ones with the actual process that was used in the original filter.

Every engineer who designs a new filter plant incorporates something that is not common to others. The outstanding features of the filter plant at Atlanta appear to be the single gallery and the open gallery as found to-day in few filter plants. The over-all filter house would come next, although that has been used in Cincinnati, Ohio, St. Louis, Mo., and Minneapolis, Minn., the last being the oldest of any material capacity. The extreme climate at Minneapolis was probably a strong argument for this type. In a southern climate, there are many features that can be adapted and incorporated that would not always be successful in the more severe northern climate. One of these is the concrete wash tank.

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* This discussion (of the paper by the late Paul H. Norcross, M. Am. Soc. C. E., presented at the meeting of the Sanitary Engineering Division, Atlanta, Ga., April 10, 1924, and published in August, 1925, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† (The J. N. Chester Engrs.), Pittsburgh, Pa.

The speaker's early experience convinced him that the inherent principle of filtration was the balancing of the water between the coagulating basins and the filter tubs to produce an unrestricted slow flow from basins to filters and thus preserve the floc already formed. This question has been discussed with the officials of the City of Norfolk, Va., for whom the speaker has recently done some work. There the water is pumped from the coagulating basin by centrifugal pump to the filters and the officials are insistent that that does not prevent good filtration. When a new filtration plant was built, however, that feature was not incorporated in it.

The speaker has never before seen any plant in which the indicator on the wash line was calibrated to indicate the inches of rise of wash water in the tub. This feature is to be commended highly as it shows the extent of the rise, and, consequently, the rate of washing.

One feature of the pumping station that impressed the speaker was the use of centrifugal pumps on direct pressure where there must be at least a variation of 50% in demand. A basic principle of that type of pump is that they are most economical when discharging against a fixed head. If the demand is not equal to the capacity of the pump, it must increase the head and, therefore, affect the economy. For this reason it will be more interesting to learn what the operating results of this plant prove to be, as compared with others, than what test duty these pumps produce, because when the machine is being tested it must be operated under the conditions on which the guaranty is made, regardless of the subsequent operating conditions. No manufacturer could compete on anything except plainly stipulated conditions. That, therefore, is a feature of test conditions to which engineers have to defer and await the results after a number of years of service.

NISBET WINGFIELD,* M. AM. SOC. C. E.—The water for the supply at Atlanta is taken from the Chattahoochee River (about 7 miles from the center of the city) and pumped to two raw water reservoirs in the city; from these it flows by gravity through the coagulating basin and filters to a clear water basin, and thence to the pumps supplying the city. This basin, called the Hemphill Station, is 100 ft. below the high points in the distribution system. The pressure on the mains is maintained by the pumps at all times, there being no service reservoir. The lift from river to raw water reservoirs is 227 ft. From the Hemphill Station it is 100 ft. plus the pressure on mains, making a total lift of 185 ft. for domestic pressure and 285 ft. for fire pressure.

Many of the problems that have confronted the municipal authorities in maintaining an adequate water supply in Atlanta are similar to those arising in every growing community, but the growth of Atlanta, both numerically and industrially, has been excessively rapid in the last fifteen years, so that the situation at times has been acute. This applies to schools, transportation, water-works, and every other public service, and means that an extra burden has been put on those entrusted with these public utilities. How well they have stood the test has been indicated by the fact that no serious interruptions

* Cons., Municipal and Hydr. Engr., Augusta, Ga.

of the service have occurred, although at times the situation must have been very trying.

For years Atlanta has been outgrowing its income, that is, the needs of the public service institutions have been greater than the funds available. An effort to catch up at least partly in the Water Supply Department, has been made by a recent bond issue of approximately $3 000 000. As explained in the paper, this was not sufficient to carry out the plans in their entirety, but has been sufficient to put the pumping, filtration, and storage capacities on a good basis, but the distribution system is not adequate by any means. New centers of distribution are being formed so frequently, on account of the building up of certain sections, that an adjustment in the pipe system is needed to meet these changed conditions, and additional funds should be provided for this purpose with as little delay as practical.

The City of Atlanta occupies the highest ground for miles, in either direction, hence there is no opportunity for a service reservoir sufficiently elevated to supply the city by gravity. Additional fire pressure in the mains (100 ft.) is not intended to furnish hydrant fire streams, its object being to force sufficient water through the small mains for use of fire pumping engines—this in lieu of larger street mains, which otherwise would be necessary.

The direct pumping and the varying pressure render it necessary to equip and maintain larger pumping units than would be necessary under ordinary conditions of service. The average consumption per day at this time ranges from 26 000 000 to 30 000 000 gal., but during the 24-hour period the rate of pumping varies from 17 000 000 to 50 000 000 gal. per day, and the pressure against which the pumps must work varies from 80 to 125 lb.

This varying condition of service does not apply, of course, from the river station to the raw water reservoir, where a constant rate of pumping may be maintained against a uniform pressure, the difference in elevation between the surface water in the suction well at the river station and the raw water reservoir being 227 ft. The difficulties which have been encountered heretofore in the operation of this station have been due to the fact that water is taken directly from the Chattahoochee River through a single conduit to a suction well of small capacity, and to the further fact that neither the 30-in. nor the 36-in. raw water discharge pipes had sufficient capacity to take care of this service alone, and an interruption of the service in either necessitated raising the pressure on the pumps above that for which they were designed, in order to overcome friction.

The trouble with the intake pipe is due to the tremendous variation in the flow of the river—the low flow being less than 400 cu. ft. per sec., whereas the flood flow is more than 100 000 cu. ft. per sec. During low flow the intake is not entirely submerged and the quantity of inflowing water is uncertain, while during periods of high water constant attention is necessary to prevent flooding through the intake well while still maintaining the proper flow to the pumps. When the impounding reservoir is constructed, as recommended in the paper, this trouble will be eliminated.

The present pumping capacity of the Chattahoochee River Station is 106 000 000 gal., and the three discharge mains have been cross-connected in

several places, so that as soon as the intake from the river is reconstructed there should be no difficulty in delivering the required quantity of water into the raw water reservoirs.

At the Hemphill Station the pumping capacity is 95 000 000 gal. per day, with a clear water basin of 10 000 000 gal. capacity from which to pump, and filters of 42 000 000 gal. capacity constantly feeding it, so that to handle this plant properly all that is now necessary is to increase the sizes of the street mains, thus reducing the pressure to overcome friction.

Probably the most interesting feature of the Atlanta Water-Works is the filtration plant, which, to say the least, is unique in that the gravity filters and pressure filters are operated in conjunction, this arrangement being made in order not to scrap the old pressure filter plant of 21 000 000 gal. capacity, which had proved through long years of excellent service that it had a right to live. Any one who has been actually in charge of the operation of a filter plant with a highly turbid raw water will appreciate the difficulty of suitably treating 30 000 000 gal. per day, having a peak draft of 50 000 000 gal. per day, with a filter capacity of 21 000 000 gal., and a clear water basin capacity of 2 000 000 gal. This was the condition of Atlanta before the construction of the filtration plant just completed.

The waters of the Chattahoochee River are, in all important particulars, similar to other rivers in the South Atlantic States, that is, the water is of good chemical quality, with few harmful bacteria, but carrying in suspension at all times large quantities of clay and silt of reddish-yellow color, varying in quantity with the different seasons. The principal work for the filtration plant, therefore, is to remove this color.

Under normal conditions the total suspended solids are approximately 43 grains per gal. From tests made by the speaker it was found that in 72 hours, with the water quiet, there was a precipitation of 32 grains, leaving 11 grains which would not precipitate. This residue of 11 grains is the exceedingly fine particles which give color to the water, so that if a settling basin of sufficient capacity is provided, the output of the filters will be largely increased. With water constantly passing through the basin the body is more or less disturbed, and precipitation will be slower. For this reason an additional length of time beyond 72 hours should be given.

The capacity of the raw water reservoirs in Atlanta is 393 000 000 gal. These reservoirs, acting as settling basins, gave sufficient time for settling, thus furnishing to the filters a water comparatively easy to treat, and making the 21 000 000 gal. capacity filter plant just about ample with a rate of filtration of 3 gal. per min. per sq. ft. At times, however, the filtered water had too much color. Many of the suspended particles giving the color are so minute that they will not be caught on the filter bed, unless the rate is extremely slow. Discolored filter water, therefore, means that the rate of filtration has been too rapid. This has now been cured, and there should be no further trouble until the rate of consumption is materially increased.

The old filter plant is interesting also, from the fact that it was the first mechanical filtration plant constructed to treat the whole supply of a municipality. The original plant consisted of twelve Hyatt filters, with combined

capacities of 3 000 000 gal. in 24 hours. Previous to this time smaller units had been installed in several places in the East, but Atlanta has the distinction of being the first city supplied through a mechanical filtration plant. These filters had two chambers, an upper and a lower. The filtering was done in the lower chamber, and when it was necessary to wash the filter bed, the sand was lifted by hydraulic apparatus from the lower chamber to the upper chamber and washed, and then returned to the lower chamber. The operation of this plant was successful, due to the fact that the water was taken from a large impounding reservoir, where previous settlement had taken place.

Several years later (1887-90), in order to lessen the original cost of filtration works, attempts were made by other manufacturers to filter public supplies without previous sedimentation, the water being drawn direct from a turbid stream and forced under pressure through filters located on the discharge pipe. Notable examples of this were: New Orleans, La., taking water from the Mississippi River, Little Rock, Ark., with water from the Arkansas River, and Chattanooga, Tenn., taking water from the Tennessee River. In each case the attempt was unsuccessful, due to the fact that silt and other suspended matter quickly formed a mat on top of the filter bed, with the result that the more pressure applied from the pumps, the denser the mat became, almost entirely stopping the flow. In several instances the filter cases were torn open by pump pressure.

These plants and many other smaller ones of which the speaker has personal knowledge, where the filter installations were similar, were later used as gravity plants, by the construction of settling and clear water basins, so placed that the pressure filter cases and pipe systems could be used without alteration, the water passing through the beds by gravity instead of under pressure. In all cases the change from pressure to gravity filters, with previous sedimentation, had the desired effect and potable water was obtained without trouble.

When the source of water supply for Atlanta was changed from South River to the Chattahoochee River, and the existing raw water reservoirs and pumping plants constructed, the twelve vertical Hyatt filters were moved from South River and set up at their present location, between the raw water reservoir and the Hemphill Pumping Station, and a small clear water basin constructed, the water passing by gravity from the raw water basin through the filters to the clear water basin, with a fall from the raw water reservoir to the clear water basin of 40 ft.

The capacity of the filtration plant was increased by the addition of twelve New York horizontal filters with a rated capacity of 500 000 gal. each, giving a total capacity of 9 000 000 gal. During the past twenty years this capacity has been increased to a total of 21 000 000 gal. This plant worked satisfactorily until the consumption exceeded the plant capacity. In connection with the new plant just completed, it will no doubt continue to give excellent service, provided the raw water is given not less than 72 hours of sedimentation, and the rate of filtration does not exceed 2 gal. per min. per sq. ft. of filter bed.

EXCESS CONDEMNATION IN CITY PLANNING

A SYMPOSIUM

Discussion*

By Messrs. Horace H. Sears, Frank O. Whitney, Andrew Wright Crawford, and Harold M. Lewis

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Horace H. Sears,† M. Am. Soc. C. E.—The speaker proposes to bring to the attention of engineers certain facts which relate to matters of design and construction where excess condemnation forms a part of eminent domain proceedings discussing the several matters under various headings, as follows:

Municipal Use of Power of Excess Condemnation.

Public Utility Use of Excess Condemnation.

Excess Condemnation Compared with Zoning Laws and with Police Power.

Financial Considerations: (*a*) Special Assessment; and (*b*) Issuing Securities.

The municipal engineer should have a clear understanding of these topics as his work of preparing the preliminary plans is one of the first matters of procedure. Given a valid law under which to proceed, the engineer plans his improvement district; he describes it definitely by metes and bounds, and thus takes the initial step. The laying out of a proposed street on an official map has been held not to be a taking to divest or impair the owner of the title. (See McQuillin Mun. Corp., Em. Dom. Sec. 1471; citing N. Y. C. R. Co. *v.* State, 55 N. Y. S. 685; 37 App. D. 42; affmd. 165 N. Y. 658; and 59 N. E. 1130; also, similar holding in *re* Phila. Pkway, 250 Pa. 257; but see case of Daley *v.* St. Paul, 7 Minn. 390, as limiting the act of filing plans, etc.)

The municipal engineer in co-operation with the City Attorney, should prepare the plans and the description characterizing the construction work proposed. The "public use" which forms the basis for eminent domain proceedings must be shown by engineering description in specifications and plans as a necessary part of proposed excess condemnation proceedings. The importance of excess condemnation is an engineering factor which the lawyer and legislator need to have defined and described for their use by civil engineers.

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* This discussion (of the Symposium on Excess Condemnation in City Planning, presented at the meeting of the City Planning Division, New York, N. Y., January 22, 1925, and published in September, 1925, *Proceedings*), is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Cons. Engr. and Attorney-at-Law, New York, N. Y.

The proper legal status or the position of the law (valid law) cannot be insisted upon too much. That must come first. As frequently the engineering fees are to be paid from funds obtained from the ultimate financing of this project, the engineer should be certain that there is a valid law and not merely a favorable opinion of counsel. If the engineer can evolve his designs with the assistance of additional land secured by excess condemnation, it may make a material saving in cost over restricting his design to the rigid lines permitted under eminent domain powers. The engineer should make sure that there is a valid law before risking his recompense or reputation. If the attorney's procedure goes wrong, he should have known better; but any mistake will rise up to plague the engineer who has been identified with invalid municipal proceedings.

Municipal Use of Power of Excess Condemnation

This filing of plans (under a valid law) with a proper description (metes and bounds) for purposes of publication, will be incorporated by the City Attorney in the first legal procedure as a Notice of Intention. The engineer should follow this step and check up his part of this very important preliminary as to plans and description. The plans in general will be concerned with a taking of private property under eminent domain proceedings, with which excess condemnation is an integral part. For reasons set forth in the Symposium,* it is highly desirable that the engineer should have the most economical area for his design, as he most certainly will be limited to the minimum expenditure for construction purposes.

This matter of economy of design and lowest cost of construction is so related to "public use and necessity" as to give an added argument for the use of excess condemnation. Where a design for construction can show material savings by the use of excess condemnation it may well be argued before the Court that boundaries limiting the area needed for construction are to be determined by "cost" and not by the minimum limits within which skillfully designed construction may be completed; but this is a fair question for the Court to decide and more properly belongs to a discussion of the legal phases of excess condemnation.†

Excess condemnation has received special attention from legislative action as to streets and parks. It should be noted that special legislative action is limited and does not give powers outside or apart from itself. There is also the need for engineers to consider whether the State Constitution permits this legislative action of a special kind. Many legislative enactments have been held invalid when tested by Court decisions. Frequently, a so-called "friendly suit" is brought to test a particular situation where untested laws are the basis for special assessment procedure.

The need of excess condemnation in connection with City Planning applies to the entire schedule of public improvements wherever the power of eminent domain is exercised. Streets and parks usually receive attention, but water,

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, pp. 1415-1452, inclusive.

† See "Financial Considerations", p. 1866.

gas, sewer, and service mains also require consideration when the matter of economy in engineering design calls for additional land adjacent to the minimum area which present methods under eminent domain allow the engineer to use.

Decisions on municipal uses of the power of excess condemnation are very limited. These decisions have been summarized,* as follows:

Unfavorable Decisions.—

(*A*) The "public use" for which private property may be taken by eminent domain means "use by the public", which is invalid when the city plans to dispose of the excess property later. (Penn. Mut. L. I. Co. *vs.* Phila., 242, Penn. State, 47.)

(*B*) Without relying on the "use-by-the-public" rule, the promoting of commercial and industrial development (that is, the creation of business thoroughfares) is not a purpose which is "public use" in the sense of justifying the condemnation of land or the use of taxation. (Opin. Just., 204, Mass., 607; 209 Mass., 616.)

(*C*) The actual result of excess condemnation, whatever may be the motive back of its use, is the taking the property of one man by eminent domain and transferring it to another. This is violative of the spirit if not of the letter of the Constitution, etc. (The Matter of Albany St. 11 Wend., 148; Dunn *vs.* City Council of Charleston, Harper's Law Rept. (S. C.), 189.)

Judicial Approval of Excess Condemnation.—

(*D*) A taking by excess condemnation involves the taking of private property by eminent domain proceedings for educational and esthetic purposes constituting a "public use". (Penn. Mut. L. I. Co. *vs.* Phila., 22 Penn. Dist. Repts. 195.)

(*E*) The condemnation of remnants of land is an exercise of the right of eminent domain for a use necessary and incidental to a public purpose. (Opin. Just., 204 Mass., 616.)

To the engineer engaged in the preparation of plans for municipal improvements, the foregoing may seem confusing and very unsatisfactory; but these legal distinctions must be understood as controlling in the States where they were rendered. Mr. Williams† states the most reliable rule as to the valid features by which engineers may work, as follows:

"Whatever doubt there may have been with regard to the validity of the statutes of excess condemnation, under the State Constitutions as interpreted by the State Courts, there can be no doubt with regard to the validity of the constitutional amendments, which can be challenged only on the ground that they are contrary to the Constitution of the United States as construed by the Supreme Court."

The need and scope of the municipal use of excess condemnation may therefore be as extensive as that of exercising the power of eminent domain, as will be developed under "Public Utility Use of Excess Condemnation".

* "Excess Condemnation", Robert Eugene Cushman, 1917, pp. 305–306.

† *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1418; see, also, Williams, "The Law of City Planning and Zoning", Chapter III.

Public Utility Use of Excess Condemnation

This phase of City Planning is entitled to similar considerations of a valid law as a basis for procedure as already discussed.

A separate consideration is needed for those cases in which the city acts as a corporation to furnish water, gas, or electric service—and sometimes transportation—to the public as an incident of its charter rights. The bona-fide public service corporation is the other basis for special consideration in exercising the sovereign right of eminent domain and its attendant powers of excess condemnation.

Relative to special legislative enactment, take the Rochester, N. Y., case cited by Mr. Vedder*:

"*Sec. 2.—Corporate Powers.*—The city has power * * * to take more land and property than is needed for actual construction in the laying out, widening, extending or relocating parks, public places, highways or streets, provided, however, that the additional land and property so taken shall be no more than sufficient to form suitable building sites abutting on such park, public place or street; * * *."

Assume that the engineer has under consideration the design for a trunk-line sewer, requiring the exercise of the power of eminent domain and the use of a street, which must be widened in connection with this improvement. Can this special statute amendment of the Rochester City Charter of 1921, made in accordance with revised Constitutional provisions to authorize "excess condemnation", be considered as a valid law for this sewer construction? Will this statute relating to city powers be strictly construed, or can several additional matters be saddled on a proposed street improvement, the said street improvement being clearly within the statutory powers?

It is not proposed to discuss here the legality of the legislative enactment mentioned. A word of caution may be in order, as all legislative enactments, until approved by subsequent Court decisions, are of doubtful value; hence, the so-called friendly suit, which most quickly determines specific points of legality with respect to a particular construction procedure.

The point under discussion is that of saddling an otherwise perfectly valid street proceeding with additional construction work. If excess condemnation is valid for necessary street widening, it would seem to follow that the engineer may introduce at the same time other construction as water, gas, electric, and even a street railway, the total cost of all this construction to be carried by the same bond issue that is the source of funds. This shows the imperative need for excess condemnation accompanying eminent domain proceedings and for a constitutional provision in every State, which will permit (and thus leave unquestioned) the validity of engineering designs based on the properly safeguarded use of excess condemnation.

Public utilities are entitled to utilize the powers of eminent domain. The attendant use of excess condemnation is of practical importance to the engineer when designing construction within city limits as well as outside them. Once the door to the validity of excess condemnation for specific use of streets and parks is opened through constitutional amendment, the same treatment

* *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1420.

must logically follow with respect to public utilities. The use of excess condemnation can be based on the cost of construction as well as on the area. The area necessary for "public use" has been the basis for eminent domain proceedings. The cost of construction is of the essence of "public use", and is frequently a greater deciding factor than area when public improvements are considered. Excess condemnation is a logical development of the use of eminent domain. It is an agency incidental to the proper "public use" of eminent domain. Why should not the principle of "agency" as recognized by the Courts be considered here. That what one may do himself, he may do through the agency of another? Apply this principle to eminent domain and there results "what the State authorizes as permissible under eminent domain, may be exercised by excess condemnation proceedings." There is constitutional authority for eminent domain—the time has arrived to secure constitutional authority for this very necessary agency of eminent domain—the power of excess condemnation.

Excess Condemnation Compared with Zoning Laws and with Police Power

The topics of zoning and police power may be properly considered together as of interest to the engineer in his program of municipal improvements wherein excess condemnation has its part. Zoning laws should be understood as forming a special feature of the police powers which the State delegates by implication and by express authority. The city charter derives its express authority from the State; but this express authority carries with it certain implied (unwritten) powers which are reasonably incidental to those powers specifically set forth. One of the most important of these powers is that which restricts absolute free personal action so that the liberty of all may be assured. This assurance of liberty in a shape that will result in the greatest good to the greatest number is a form of police power and is concerned with public safety as much as the matter of morals and health is incidental to public safety.

Zoning laws have recently come into use and have been upheld as a valid exercise of the police power. Claims have been made that the power of eminent domain was being exercised and that compensation was due an owner where damages resulted. A recent police law of special legislative design affecting New York, N. Y., is that of the so-called "Rent Laws" which have undoubtedly, in some cases, arbitrarily impaired contracts made between landlords and tenants. Even this extreme and valid exercise of the police power is of interest to engineers. However, it is the zoning law that the engineer should study. The application of the principles of zoning by the engineer when the plans for sub-division of territory are in their formative stage, offers great possibilities, which are independent of the use of excess condemnation powers.*

The selection of zoning laws in developing new areas gives partial assurance that in the future excess condemnation will not be as necessary as formerly. The engineer should clearly understand the fundamentals which distinguish zoning from excess condemnation; and also the relation which each of the following terms bears to the other, namely, excess condemnation,

* "The Law of Zoning", by H. S. Swan, and "Zoning", by Edward M. Bassett, National Municipal League, New York.

eminent domain, zoning, and police power. This is of importance to the municipal engineer as his preliminary designs are intimately concerned with one or all. The legal viewpoint on these subjects is quite different from the practical application which the engineer must make of them.

The building laws are typical of special police power ordinances passed by a city, within its charter powers. The restricting of areas for special purposes so as to safeguard and promote the public health have generally been held valid, but many instances have occurred where zoning laws have been held invalid. The engineer, therefore, is interested in knowing whether the procedure under which he works is a law for zoning or for excess condemnation.*

The use of zoning laws under the police powers should be clearly understood as to their legal and practical distinction. The powers of excess condemnation are more closely related to the powers of eminent domain. Each of these last two powers is found where there is a condemnation without the owner's consent (a taking of realty and the incidents connected therewith) and a payment to the owner for his property thus taken by virtue of the sovereign right to take for "public use".

Financial Considerations

Remnants of realty and gore strips of unusable area form a costly part of the use of excess condemnation. Mr. Vedder† gives a condensed statement of the condemnation of excess area and the sale of part. This feature has been discussed in certain cases already cited. (See 11 Wend. N. Y. 148; Harper's Law. Rep. (S. C.) 189; 204 Mass. 607; 242 Penn. St. 47.) The available decisions on excess condemnation treat this practice as unconstitutional in the absence of express constitutional amendment making such use valid.

Cases where the right of exercising the power of eminent domain requires a right of way as an easement over the land of another and the boundaries are clearly defined, illustrate the application of the principle of "minimum area for a public use" and clearly of "reasonable necessity". It came within the province of the civil engineer to determine and describe the area for a highway —its length and breadth—and to file an accepted plan. When embankments became necessary and the foot of the slope (or top of the cut) made an increased width necessary—as compared with the width of actual traveled way—this constituted in reality the first departure from the "eminent domain" area actually needed for the traveled portion of the highway. The excess spread for the slope was excess area if it be conceded that a vertical wall retaining the traveled width of the roadway could have been erected.

As a matter of practice many engineers have sanctioned the additional expense of building a retaining wall to avoid condemning a remnant or gore piece of land because the owner would secure damages for his injury out of all proportion to the proposed use. Eminent domain proceedings would have been allowed but their excessive cost justified the engineer in neglecting the minimum construction cost in favor of keeping off additional land. It is sub-

* "Zoning and the Courts in Texas", *National Municipal Review*, June, 1924.

† *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1421.

mitted that, the principle of eminent domain being admittedly the seizure without consent and payment therefor to the owner, the same principle should obtain in the taking of excess land without consent and payment therefor, the use of the excess land being based on financial considerations within the scope of the engineer to determine; and dependent for its supporting facts on the engineer submitting to the same tribunal that passes on eminent domain proceedings the cost data and valuations which form the basis for the valid eminent domain procedure.

This "financial" element of the cost data is the criterion and not the area. "What does the public pay?" is as pertinent to decide as, "What does the public acquire in land area?" The public is burdened for years with a bonded debt, long after a suitable use is made of the excess area. Let the engineer approach this need of excess condemnation as he would the plans for an earth fill for a highway; let him file his preliminary plans for eminent domain based on the "cost" of the proposed project, and make the condemned area include all that is reasonably incidental to a "public use" and to the lowest total cost to the taxpayer. The engineer's estimate will include, of necessity, future transactions in land, but it also includes costs of future construction and the purchase of materials. Land is merely an additional element to the customary items of labor and materials. The cost of land, labor, and materials should be recognized by the Court as equally applicable in determining the reasonably necessary cost to the public. This is submitted to engineers, because they constitute the proper source of initiating proceedings. They will have to work in harmony with the City Counsel.

Changed conditions demand changed methods. A change in the law has been refused, therefore it is necessary to change the method of applying valid laws. This matter of applying new methods will not meet with favorable reception in the older cities, especially where decisions have been handed down which have rejected the method of reducing costs by selling "remnants" of land resulting from excess condemnation proceedings. There is a possibility of using "cost to the public" in those jurisdictions where constitutional amendments have allowed excess condemnation proceedings.

Given a constitutional amendment allowing the use of excess condemnation proceedings, the next step is to limit its application. By showing on the procedure plans filed for publication the total area affected by eminent domain plus an excess area which is "reasonably necessary", a rational claim would be placed before the tribunal which will consider the objections of owners and citizens.

Financial considerations also enter into the use of excess condemnation by way of paying the cost, initial and final, of any given municipal improvement. The engineer is interested in this feature of the "source of funds", be it the general city fund, the special fund created by ordinance, or the special assessment tax on the property benefited. This opens a field for discussion which is here arbitrarily confined to (*a*) special assessment financing and (*b*) issuing securities.

Engineers who are not acquainted with methods of financing the projects on which they somewhat blindly follow political activities may well be careful.

The reputation of an engineer is in part related to the success or failure of the projects on which he works. Failure of a splendid engineering project may frequently be laid to the fact that it was not financed properly or to invalid proceedings—that is, invalid from the start and not detected until some months after the project is in full swing. Special assessment proceedings are not difficult—merely routine—but frequently mistakes are made by ignorant public officials. The engineer should verify these proceedings as his initial fees and his ultimate recompense are often dependent on their validity. Given a valid law under which to use the power of excess condemnation, the construction work will frequently follow procedure known as "special assessment".

Special Assessment.—The details of special assessment procedure do not come within the scope of this discussion, except as it is based on excess condemnation, which calls for financing by a district specially benefited. The Southern and Western States are more inclined to handle special assessment district procedure than the Eastern cities. Whenever possible, the engineer should have the general credit of the city as an underlying promise on all special assessment financing. When securities are issued which are merely a lien on the abutting property of a special assessment district, a greater price must be paid for the use of money, as these securities are considered of inferior quality compared with those that have the general credit of a city supporting them. (See Fla. Laws, 1923, Chapter 9298 (Spec. As. Bonds as General Obligations of the Municipalities).)

Issuing Securities.—The engineer should have a good understanding of the auditing and financing of the projects which he initiates. Given the organized engineer office of to-day, with subordinates capable of handling routine engineering matters, the chief engineer should study the related legal and financial questions which may make or break his project.

The securities that must be issued to pay for the excess condemnation procedure give one cause for thought. Shall the entire cost of the project—direct costs, eminent domain cost, and excess condemnation costs—be treated separately? If the entire budget can be handled as one item it will certainly simplify matters. Then comes the very usual and vital question—does an invalid excess condemnation item, which has been grouped with valid items, invalidate the entire procedure? This is a point worth considering. The answer depends on the project and all the surrounding circumstances.

The total amount involved is some indication of the cost of handling excess condemnation. Anticipated profits from realty transactions are not always realized; no more than where a private operator in real estate is concerned. If the municipal operations in the sale of remnants from excess condemnation proceedings fare no better than the average results from municipal operation of public utilities, there will be little for the profit side of the ledger. Municipal activities had best be restricted to acting as an agency of the State. At the same time, the new, rapidly growing communities of the South and West have a legitimate use for municipal ownership of public utilities, because no one else will help activities that have still to demonstrate their value. Thus excess condemnation proceedings are most likely to be favorably received in

those States where the Courts have not taken a decided stand against their validity. The full faith and obligation of the city should be pledged to support the securities issued to finance excess condemnation. The engineer should see that this is done in every case. The type of security, whether notes or bonds, will depend on so many factors that only the character of the lien supporting the loan can be mentioned. The very highest kind of financial obligation should be given by the city to support excess condemnation financing.

To the engineer is given an opportunity to study excess condemnation procedure as affecting his designs and as the proper auxiliary of eminent domain proceedings. The consideration of the legal, engineering, and financial elements only serves to emphasize the importance of this subject and to indicate the large amount of educational work which must be done before all the States will grant proper constitutional amendments placing excess condemnation on a par with eminent domain proceedings. The economic use of remnant land for the furtherance of the public welfare provides an opportunity for the engineer to assist in the education of the public concerning the powers of excess condemnation as a saving in cost to them on numerous improvements.

FRANK O. WHITNEY,* M. AM. SOC. C. E.—The subject of excess condemnation has been very carefully studied by the Street Laying Out Department of the City of Boston. Before it was enacted, the Massachusetts law was expected to open a way to secure much benefit esthetically, and, in some cases, to become a source of much profit. Since the enactment of the law making it possible to take excess quantities, its application on a large scale has been considered in numerous cases of proposed thoroughfares.

Generally, it has been deemed to be impracticable on account of the great value of the properties concerned, of their present complete development, and of the necessity of placing high values on the property to be sold after the condemnation. In only one instance has the Legislature been asked to give its approval to the use of the law, namely, the laying out of Stuart Street in the Back Bay. Although the Legislature complied, the street was laid out and constructed under the general statutes, as the City Council failed to accept the Special Act.

This is one instance where excess condemnation might have proved beneficial, as the street (about a mile in length) was laid out over run-down or stagnant property the value of which before the improvement had been assessed at from \$3 to \$12 per sq. ft., and was either vacant land or occupied by ancient and cheap buildings. The improvement has resulted in a greatly increased value to abutting property, as evidenced by recent sales averaging from \$20 to \$25 per sq. ft. As the City of Boston is allowed to assess betterments wherever sustained up to the cost of the improvement, the seeming loss of opportunity in this case was not as great as it would at first seem, the cost (\$3 100 000) being offset by a betterment assessment of \$2 700 000.

Several propositions for an adequate thoroughfare connecting the North and South Stations have been suggested and estimated, which as yet have

* Boston, Mass.

not materialized. Excess condemnation has been considered in these estimates, but in each case on account of the high value of the properties involved, it has not been used, as the remaining land would have had to be sold for three times its present value to provide a profit equal to the cost.

In the case of a large improvement a considerable amount of property would be temporarily untaxable. The carrying of this real estate would require large capital with heavy interest, and also involve the city in an immense real estate business.

The Massachusetts law necessitating a special act of the Legislature in each case of the application is very inconvenient, as comparatively few cases arise when the Legislature is in session. The general assessment laws provide ample means for such compensation as is necessary in most instances.

Excess condemnation may be of great benefit in special cases, but should not be expected to provide a universal remedy for all the ills involved in laying out streets. If the City of Boston could have a law that would allow the taking of small remainders at will, much good would ensue, as abutters could then be protected from the avarice of small owners, and unsightly indentations would be avoided.

ANDREW WRIGHT CRAWFORD,* ESQ.—The principle of remnant condemnation, which is restricted excess condemnation, was applied in the case of Fairmount Park in Philadelphia, Pa., about 1870 and proved very valuable.

In the case of the King's Highway, in London, where a street 100 ft. wide was extended through a closely built-up area, the cost of the entire outlay for the right of way and for the abutting property was about $30 000 000. None of the abutting property was resold; it was leased on 100-year improvement leases. Capitalized on the basis of 4%, the entire cost has been recouped, according to the latest information.

It is necessary to "think through" in such matters, in the case of this London example to the end of 100 years, when the municipal corporation will own the enormously valuable property on both sides.

One of the greatest advantages of excess condemnation is that in the re-sale of the excess property, or in the leases of such excess property, on a 100-year improvement lease, it may be provided that whatever buildings are erected must be approved as to design by the Municipal Art Commission. In the long run that will be a very powerful factor in beautifying the city, especially on the wider and longer streets.

Excess condemnation ought frequently to be used in the case of remnants fronting on streets. Mr. Leavitt† instanced a small strip, 3 to 20 ft. in width, running along the street for some distance. The trouble there was in the lack of education of the engineer. There is no reason why any street intended to be about 100 ft. wide, should be exactly 99 ft. 12 in. wide all the way through. For a block, it might be 97 ft., or 103 ft., or even 110 ft. It is well enough to have straight streets, but as to their boundaries, that is absurd. In

* Philadelphia, Pa.

† *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussions, p. 1425.

the case of small remnants that are parallel or slanting at a slight angle to the street, the real trouble is with the inflexibility of the particular engineer who was not sensible enough to vary his street somewhat from block to block, or within the block.

HAROLD M. LEWIS,* M. AM. SOC. C. E. (by letter).†—The advantages of excess condemnation have been very well summarized by Mr. Leavitt.‡ It may seem strange that a policy which has so many arguments in its favor and is generally admitted by students of city planning to be equitable and advantageous should meet with so much local opposition when attempts are made to utilize it. A brief summary and analysis of the reasons for such opposition may be of interest. The principal ones can be stated as follows:

1.—The term, "excess condemnation", which is an unfortunate one tending to arouse immediate opposition.

2.—Objection to any change in old and well-tried methods.

3.—An increased first cost of improvements.

4.—Reluctance of local property owners to relinquish the opportunity of making large speculative profits from holdings immediately adjacent to the improvement.

There are probably no two words the use of which is more apt to have the effect of waving a red flag in the face of public opinion than "excess" and "condemnation". The former implies that more property is to be taken than is necessary and the latter implies a confiscation of private rights. The term, "Remnant Act", used in Massachusetts, was much less inclined to arouse opposition, but the acquiring of remnants did not go far enough to bring about the best results. The writer does not attempt to suggest a better term, but if one could be found which would refer only to the taking of remnants and a re-subdivision of abutting property, it would certainly be advantageous. Provisions for purchase or condemnation of adjacent parcels necessary to make an effective re-subdivision of land fronting on the new street or park improvement properly belong in the text rather than in the title of the legislation.

Any change in customary methods will always meet with opposition but this can be overcome through courageous perseverance by progressive citizens, if they are properly enlightened by the professional men studying the problems.

A method of decreasing the first cost of bonding the excess lands acquired and exempting such bonds from the debt limit has been suggested by Mr. Leavitt. There is no doubt that such bonds would prove safe investments, as in many cases the resale of the land outside the limits of the improvement should provide a substantial profit. By this means a fund might be created which could be utilized to assist in financing other similar improvements needed in the city.

Some street-widening improvements result mostly in local benefit but any large scale improvement is greatly to the advantage of the whole city and

* Executive Engr., Regional Plan of New York and Its Environs, New York, N. Y.

† Received by the Secretary, September 26, 1925.

‡ *Proceedings*, Am. Soc. C. E., September, 1925, Papers and Discussion, p. 1425.

raises land values several blocks distant from the actual improvement. If the project is the cutting through of a new and important traffic or transportation thoroughfare its influence may be felt to still greater distances. It is obvious, therefore, that many property owners and the city at large could profit by improvements which may be held up by the selfish ambitions of those few whose property might be taken, although they would receive a fair value in return. The majority should rule.

THE ST. LAWRENCE WATERWAY TO THE SEA

Discussion*

By Messrs. Gardner S. Williams, E. P. Goodrich, David B. Rushmore, Maurice W. Williams, L. H. Hart, A. Lindblad, and Walter M. Smith.

Gardner S. Williams,† M. Am. Soc. C. E.—The speaker is in harmony with the general expressions which have been made in the discussion of the Great Lakes-St. Lawrence Waterway. Referring to the remarks‡ of Mr. Sabin, however, he would call attention to certain historical facts. The early explorers coming up the channels connecting the Great Lakes reported that those channels varied in depth from 8 to 12 ft. Mr. Sabin has stated that there is now a 21-ft. channel through the same reaches. Students of hydraulics can readily decide whether the deepening of channels that were between 8 and 12 ft. to 21 ft. in depth has or has not enabled more water to flow out of the Upper into the Lower Lakes.

Attention is directed to the program which was recommended in the report on the St. Lawrence Waterway submitted to the International Joint Commission regarding the development of that section of the river below the International Boundary. As stated in the paper, that program contemplates bypassing the traffic through 28 miles of canal and five locks, counting the twin locks as one. It contemplates no power development within that reach, although there are about 2 000 000 potential h.p., and is estimated to cost $92 000 000.

It has been intimated that the reason the Commission adopted its method of treatment of this section of the river was the belief that there was not a market available for that amount of power in connection with what would come from the upper reach. In that same report, however, the Commission estimated the horse-power demanded within 300 miles of the Long Sault to be 13 350 000 e. h. p. in 1925, for the United States and Canada, and, in 1930, 15 375 000 e. h. p. is estimated, or an increase in 5 years of 3 025 000 e. h. p.§

The contemplated works will not be built in five years. If the estimated rate of increase of power demand continues, according to the Commission's own figures, that power would all be absorbed as soon as it could be put upon the wires. Referring now to the super-power report, which was an investigation of the demands for power in the region of the United States from Washington, D. C., on the south to Boston, Mass., on the north, the central part of this zone

* Discussion of the paper by Francis C. Shenehon, M. Am. Soc. C. E., continued from October, 1925, *Proceedings*.

† Cons. Engr., Ann Arbor, Mich.

‡ *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1706.

§ Senate Doc. No. 114, 67th Cong., 2d Sess., p. 175.

is within reach of the St. Lawrence power.* That report states† that the energy economically to be taken from the super-power system in New England and New York and Upper New Jersey—all of which territory is within 300 miles of the generating points of the possible power developments of the Great Lakes-St. Lawrence Waterway—in 1930 from new sources will be 2 232 000 e. h. p.; that is, it is estimated that in 1930 there will be needed from a super-power system in this district new power to the extent of 2 232 000 e. h. p. It was further estimated by the Super-Power Commission, that the increase in steam-generated power in the area noted, from 1919 to 1930, would be 2 059 000 e. h. p.‡ That is about all the St. Lawrence River would yield if distributed over the wire, into that territory. It appears, therefore, that the idea that there is no use for the power—that it is not needed, and could not be disposed of—is a mistake.

The development of this water power will be a source of revenue to Canada, and a source of revenue, through sub-station service, to the people of the United States.

There is, however, another element involved. The people in the Central States are not very much interested primarily in the development of water power to be used in New England and Northern New York; but a certain amount of power is to be absorbed in that territory, and whether it comes from the St. Lawrence and Niagara Rivers, or from the burning of coal, will make no difference. The power will be utilized. Now, if that power does not come from the St. Lawrence River, or from the Niagara River, and some minor water-power sources that are hardly worthy of consideration in connection with them, it will have to come from coal. The amount of fuel consumed by public service corporations in New England, New York, and the northeastern part of Pennsylvania, for 1919 was more than 11 000 000 tons of coal; or, more accurately, it was equivalent to more than 11 000 000 tons of coal. Some of this fuel was oil and some of it was gas; but the bulk of it was coal. The coal used by public service corporations in the States of Illinois, Indiana, Ohio, Michigan, Minnesota, and Wisconsin for the same period was about 10 700 000 tons.§ In other words, if sufficient St. Lawrence power went over into New England, New York, and Northeastern Pennsylvania to replace the coal that is now burned by the public utilities in that district, there would be sufficient coal released to supply the public utilities in the States of Illinois, Indiana, Ohio, Minnesota, Wisconsin, and Michigan, and that is why the people of the Central States are interested in the utilization of St. Lawrence power in New England and New York.

Simply as a matter of comparison, the total output of fuel for the United States is the equivalent of about 600 000 000 tons of coal per year; and the railroads use 180 000 000 tons. The Super-Power Commission previously referred to, in investigating the cost of the operation of railroads by means of central sta-

* "A Superpower System for the Region between Boston and Washington", by W. H. Murray and others, *Professional Paper No. 123,* U. S. Geological Survey, 1921.

† *Loc. cit.,* p. 181.

‡ *Loc. cit.,* pp. 160–161.

§ U. S. Geological Survey, Monthly Reports of Fuel Consumption.

tions and electrical locomotives, showed that nearly 75% of the fuel burned by the railroads could be saved if they were operated from central generating steam stations.* If the railroads in this district are supplied with electricity generated by water power, there will result a further saving, which becomes of interest.

Turning now to another phase of the subject, it has been stated that the estimates for this project were probably inadequate. The speaker most fully concurs in that opinion. He has taken some pains to compare the costs of similar works where such have been built under the direction of the Corps of Engineers of the United States Army as far as similar work can be found, and from the records, his estimate, or his guess, is that the cost of this development, including the water power features, will be more nearly $600 000 000 than $500 000 000, which latter sum is about twice as much as the last report indicated would be involved.

Opinions have been expressed as to whether water power should build this waterway, or whether water power should be subsidized, or whether water power should be considered at all, or whether navigation should be considered at all, and perhaps the speaker may also be permitted to express an opinion, that is, the water power of the St. Lawrence River is needed now, and can be absorbed as soon as it can be put upon the wires. The water power development of the St. Lawrence River will provide almost everything that navigation needs except the locks and the deepening of the channels at the upper ends of the pools. Therefore, let the water power interests build the dams and develop the water power as fast as they will, under the most liberal terms that the Governments can grant them, and let the Governments provide the cost of constructing that part of the work which is necessary for navigation. If the problem is developed in that way, the burden will not be so great but that Canada, even with the enormous load that she is carrying, can take a reasonable share in this project. The people of the United States can hardly realize what conditions must be across the border, but if they examine the expenditures of Canada during the World War, and the vast projects in public works which Canada has been carrying on, they cannot blame her for hesitating to bind herself at this time to the expenditure involved in the development of the St. Lawrence Waterway. If that cost can be put where those who will profit the most by it can bear it, however, the cost will be reduced, as far as the Governments are concerned, to a figure that seems to be entirely within their reach.

Next to the Great Lakes-St. Lawrence Waterway, there is no subject before the people of the United States to-day about which a greater amount of misinformation has been disseminated than the so-called Chicago Drainage Canal, and although the Canadian Government—and this is said not in any spirit of antagonism or of criticism—is demanding, or at least suggesting, that the Chicago diversion be prohibited or restricted, attention should be called to the fact that in a treaty which now exists between the United States and Great Britain, covering the use of the boundary waters,

* Super-Power Report, *Professional Paper No. 123*, U. S. Geological Survey, p. 62.

there is a provision that the Dominion of Canada may have 16 000 cu. ft. per sec. more water to use at Niagara than the United States. If one will go back to the records, and those records are in Canadian publications, it will be found that such allowance was made to compensate Canada for a diversion supposed to be 10 000 cu. ft. per sec. at Chicago.

Attention may also be called to the fact that, whatever the effect of a diversion of 10 000 cu. ft. per sec. on the navigable depths of the Great Lakes may be, the proposed diversion of approximately 15 000 cu. ft. per sec. through the Chippewa Canal, around Niagara Falls, if uncompensated, will lower Lake Erie by approximately 50% more than the diversion at Chicago; and that the lowering of Lake Erie will be accompanied by a lowering of Lake St. Clair and the Detroit River, of approximately the same amount as that due to the Chicago diversion, and by a lowering of Lakes Michigan and Huron by approximately one-half the latter amount. Attention is further called to the fact that Canada is diverting about twice as much water through the Welland Canal at present as was so diverted prior to the World War. When Canada contends that the diversion at Chicago is lowering the Great Lakes, and wants it stopped, it may be proper to suggest that consideration be given to the effect of its own diversions on the levels of Lake Erie and the Upper Lakes.

The speaker does not oppose diversions of water through the Canadian developments for water power below Niagara. He considers it more economical to take that water through the Queenstown-Chippewa Canal to utilize the large head that is possible by obtaining it near Lake Ontario, than to take it at the Falls and derive only one-half the power. He is in favor of all that, but let there be fairness to all sides.

E. P. GOODRICH,* M. AM. SOC. C. E.—It is obvious that Government expenses for deeper waterways and for harbor improvements are subsidies of one kind or another. The speaker would suggest another kind of subsidy for consideration at least by the present International Joint Commission, and possibly by the board or commission which Mr. Parsons has suggested† might be appointed as an auxiliary investigating body. This alternative is a railroad subsidy to cover what might be called full train operation through from origin on one of the Great Lakes ports to some seacoast port, on an export tariff basis. It might be handled, for example, by a Federal commission, like the Wheat Commission, which would provide funds for by-passing the congested railroad throats on the railroad systems, as they exist to-day, so that the operation of a through train could be expedited and thereby decrease the time required of freight cars, the extra return accruing to the railroads, which, in their turn, could distribute this saving, at least in part, to the shippers through a lowered export rate.

Certain investigations made in connection with the project for deepening the Upper Hudson to provide a Troy-Albany Port gave statistics which may be of interest in this connection. The present probable total export Gulf and Pacific commerce, to and from the district that was tributary to the St.

* Cons. Engr., New York, N. Y.

† *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1694.

Lawrence, as well as to the Upper Hudson port, was carefully estimated on several bases. A figure of 15 000 000 tons per annum, as a total of all types, was finally reached. Obviously, a ship will not come inland, making a special trip, unless it can have a large proportion of its cargo assured. In other words, there must be facilities at any lake port for collecting tonnage of a considerable amount destined for one or perhaps a group of foreign ports, so that the vessel can have sufficient inducement to make the trip. Studies were made to determine as to what commodities and in what quantities there was sufficient inbound and outbound tonnage to induce a vessel to make such a trip, at least as far as the Upper Hudson port. It was calculated that a tonnage not to exceed 5 000 000 would be thus available to-day, and that at present there might be diverted additional tonnage not to exceed 1 500 000, with a possible increase to 5 000 000 in the course of the next decade, estimating on the rate of growth which has taken place in the Panama Canal, and other ship canals, and harbors in various parts of the world.

It may be of interest to discuss the eventual tonnage which may be derived in that way. Working on the basis of population estimates of the United States as a whole, and the proportion which it is expected will find itself in the tributary area of the hinterland of the St. Lawrence, say, 100 years hence, and basing these estimates on studies by Professors R. Pearl* and E. W. East,† and by the speaker,‡ it is probable that three times as much tonnage will be found available in the next 100 years. Assuming 6 000 000 tons per year, and 40 tons per car, this gives 150 000 cars per year; and assuming a 1 200-mile haul, at 4 miles per hour, 300 hours will be required. If by means of detours around traffic throats and the same methods of operation used to expedite freight as are common in expediting through traffic across the continent, for example, with reference to passenger trains, this time were to be cut in half so that an average speed of 8 miles per hour were secured, a saving of 150 hours, or 22 500 000 car-days would be made. At $1 per day, which is the usual rate, there is obviously an annual saving of $22 500 000, simply in the use of cars. Capitalizing this, and assuming that $5 000 000 is required for each throat to provide facilities to expedite travel, there still remains a saving of $162 000 000, capitalized, or $8 100 000 per year. On the basis of the 6 000 000 tons assumed, there is an immediate saving indicated of $1.35 per ton, which might be returned directly to the farmers in their shipment of grain, or to other shippers and receivers of freight, provided the plan suggested could be put into effect.

The speaker does not wish to be misunderstood as making this suggestion with reference to railroad operation. He has long been interested in the development of water terminals and waterways and believes that, eventually, the St. Lawrence project may be economically feasible, but it is obviously unwise for the two countries as a whole to expend moneys for waterways if the capital already invested in railroads might be used possibly to better advantage.

* "On the Rate of Growth of the Population of the United States Since 1790 and Its Mathematical Representation", by R. Pearl and L. J. Reed, *Proceedings*, National Academy of Sciences, Vol. VI, No. 6, June, 1920, p. 275; also, "A Further Note on the Mathematical Theory of Population Growth", by R. Pearl and L. J. Reed, *Proceedings*, National Academy of Sciences, Vol. VIII, pp. 365–368.

† "Mankind at the Crossroads", by E. W. East.

‡ In a forthcoming paper to be presented before the American Statistical Assoc.

The paper is of considerable interest because it points out some of the things which have not heretofore been discussed, and the author is to be complimented for providing the opportunity to discuss these various points.

David B. Rushmore,* M. Am. Soc. C. E.—Although forecasting economic and social conditions is only an approximation, it is certain that by the time hydro-electric development in connection with the St. Lawrence Ship Canal can possibly be completed, all the electric power produced can easily be consumed, if less than one-half of it in Canada, surely much more than one-half on the American side. There is no engineering difficulty of great magnitude in connection with the design of the power-house, of transmission systems, or of the distribution and use of this power. What is lacking at present is an understanding and an appreciation on the part of the average citizen of New England, New York, New Jersey, and Pennsylvania—the situation discussed so well by Mr. Gardner S. Williams†—of the benefits to be received by this power.

There is no man who, if he saw a coal mine on fire, would not endeavor to put it out, or to stimulate the efforts of those responsible for the fire to extinguish it. Wherever water power remains undeveloped, within reach of markets that are burning fuel, as at Niagara Falls and on the St. Lawrence, there is a coal mine on fire. At the World Power Conference, held in London, England, in July, 1924, one of the most striking features brought out was the many ways in which the people, through the Governments, are co-operating with the people as organized through private corporations. The great expenses necessary for the St. Lawrence investigation were contributed by people interested, who felt that it would be desirable to consider all its possibilities and that some satisfactory way should be evolved whereby the Government and private parties could co-operate to a satisfactory outcome.

Consider the demand for greater publicity; incidentally, that is one thing that can be done by the members of the Society. The great benefit from presenting such an interesting paper is that the public can be educated regarding the engineering facts and economic benefits to be received. Then there is a demand for a new kind of engineering—a presentation of engineering facts with as accurate a forecast of the economic and social conditions as possible, and a design of the project that is going to bring together and harmonize all the elements involved and that will permit in a reasonably short time of carrying this great project through to completion.

Maurice W. Williams,‡ M. Am. Soc. C. E.—Reference has been made to the diversion of water from Lake Erie through the Welland Canal as being greater in quantity than certain other amounts which are diverted into avenues lying wholly within the United States, such as the Chicago Drainage Canal. The Welland Canal simply happens to lie in Canadian territory, but is open to the free use of the vessels of both countries. That being the case, certainly Canada cannot properly be charged with the total quantity of water used in

* Hudson, N. Y.

† See p. 1874.

‡ Vice-Pres., Technical Advisory Corporation, New York, N. Y.

the Welland Canal. It may very properly object to the diversion of water for the use of any municipality in the United States, however desirable such diversion may appear to that municipality, the more so as all water passing through the Welland Canal is retained within the St. Lawrence drainage area while all water diverted through the Chicago Drainage Canal is permanently lost to the St. Lawrence.

L. H. HART,* ESQ.—The speaker heartily agrees with the great majority of those who are discussing the St. Lawrence Waterway project, that it is inevitable. He wishes, however, to summarize briefly the reasons why the money should not be spent during the lifetime of this generation. All these reasons fall under the general head of putting first things first.

On the navigation side of the question, the control works mentioned by Mr. Shenehon needed to raise and stabilize the lake levels, and the deepening of harbors and channels, are essential to the general plan. Let them be constructed first, as they are within the United States and will be worth many times their cost, whether or not the St. Lawrence River in Canada is developed. Let full use also be made of the existing 14-ft. canals in the St. Lawrence, the Ohio, and the Mississippi Rivers, and of the 12-ft. canal from Buffalo to New York. New York State has contributed this 12-ft. channel to the National welfare free, at a cost of $200 000 000. Many residents of the State feel that before they are asked to pay a heavy tax for new plans, the existing one should be given a trial. Some feel that the cities of Toronto, Quebec, and Montreal, would be benefited by the St. Lawrence plan, at the expense of American taxpayers, and that it would withdraw commerce from cities in New York State. The speaker does not hold this view.

At present, the bulk of traffic leaving the Great Lakes goes, not to Europe, but to the New York District. The St. Lawrence Channel and the New York State Barge Canal are now being used to only a very small fraction of their capacity. The tonnage which goes from the Great Lakes Districts to Europe could easily be carried on the New York State Barge Canal, even though it is only 12 ft. deep. In short, the time would not seem to be ripe for spending American money for improvements in any foreign country. It should be noted that six of the nine locks which are proposed on the St. Lawrence are entirely within Canadian territory.

Relative to the power side of the question: Before developing power in Canada, let full use be made of Niagara Falls. Only one-fourth of the possible power output is being developed there. The obstacle is that the scenic effect might be marred. The proposed control works, however, could easily regulate and distribute the flow, so that no beauty will be lost. At present, $100 worth of water is wasted for every visitor who enjoys Niagara. The Lower Niagara River in the Gorge is merely awaiting capital for its development. The possibilities there are very great, in comparison with those on the St. Lawrence River. There are also great possibilities in the super-power project and in the proposition to burn coal at the mouth of the mine. These are strictly American, and of known value.

* Buffalo, N. Y.

The wisdom of developing power before the market is in plain sight is very questionable, even for a wealthy Government Treasury. The Muscle Shoals Project, almost completed at a cost of $100 000 000, goes begging for lack of a bid of more than $5 000 000. If, as Mr. F. P. Williams claims,* the power industries can subsidize navigation, American and Canadian power interests will be willing to do it on their own account.

A. LINDBLAD,† ESQ.—The speaker is not a civil engineer and thus is not handicapped by any knowledge of the engineering difficulties in developing this inland waterway for navigation purposes. He will discuss this project, therefore, only from the standpoint of its shipping possibilities.

Several statements have been made regarding the kind of ships that can be used. In his discussion,‡ Professor Sadler made this point clear, when he stated that he would never expect large ocean-going vessels of the passenger type to come to the Great Lakes. Such vessels are not wanted there; but the Great Lakes should be connected with salt water, so that all those vessels which do not have too large a draft can come to the Lakes. There is no need of constructing new vessels. If a depth of 25 ft. is obtained in the channels, that is sufficient to permit approximately 70% of the world's tonnage to enter; and if that depth is increased to 28 ft., practically all ordinary cargo vessels will have access to the Great Lakes.

It has been stated§ that the cost of ocean-going tonnage is at present two or three times the cost of tonnage on the Great Lakes. That is really not correct. That it is about one and one-half times that cost would probably be a safer statement. Mr. Parsons also stated that the cost of running ocean-going vessels on the Great Lakes would be too great, partly because the vessels were more expensive in first cost, owing to the need of a surface condenser, etc., and partly because they need a larger crew than the lake freighters. It is true that they need surface condensers, but fitting a surface condenser instead of a jet condenser is really a rather small item compared with the cost of other parts of the machinery, and does not add much to the first cost. It is also true that there is a small increase in the numbers of the crew on ocean-going vessels as compared with lake boats. In other words, the cost of running ocean-going vessels is somewhat greater than the cost of running the present lake freighters, but that does not justify the statement that it would be uneconomical to use ocean-going vessels in combined lake-and-ocean traffic.

To obtain the full benefit of a deeper channel it is, of course, necessary also to have increased depth in the Great Lakes ports; but the cost of deepening these ports is not so enormously large as some people would imply. It must be remembered that the shipping on the Great Lakes is not evenly divided among the Lakes, but is concentrated and handled at only a few ports. To deepen these ports to about, say, 25 ft., will not, in the speaker's opinion, be such an enormously costly undertaking.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1701.
† Prof., Dept. of Naval Architecture, Univ. of Michigan, Ann Arbor, Mich.
‡ *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1695.
§ *Loc. cit.*, p. 1692.

In opposition to the St. Lawrence Waterway project, it has also been stated that it was not practical because it could provide for navigation during only for seven to eight months of the year. Everybody connected with shipping knows that a very large part of the world's commerce is seasonal, and the speaker believes that certainly the largest part of manufactured products could be handled easily in a navigation season of seven months. Consider a parallel from Europe. The whole region around the Baltic, including Sweden, Finland, Russia, Germany, and the new countries like Poland and Esthonia, have to be satisfied with approximately eight or nine months of navigation. Nevertheless, all these countries have been able to develop quite a foreign trade. The speaker thinks that it is an absolute mistake to say that shippers necessarily must depend on twelve months of navigation.

It has also been said, that ocean-going vessels coming to the Great Lakes would not be able to use the present terminal facilities. The grain elevators in particular are said to be suited only for lake ships and would need to be changed. That is another incorrect statement. It is true that ocean-going vessels are not constructed, like the lake freighters, with a very large number of short hatches spaced 12 ft., center to center, which hatches fit the 12-ft. spacing of the loading spouts of the grain elevators. It is not necessary to be able to load through all the loading spouts. Only those spouts which fit the hatch arrangement of an ocean-going ship need be lowered and used. It is hard for the speaker to believe that any greater changes are needed than to increase the draft at the elevators. The other changes are unimportant.

In discussions of the St. Lawrence Waterway, the speaker has heard a great many statements to the effect that salt-water vessels would not be able to navigate on the Great Lakes, but he has not yet been able to find a naval architect or a marine engineer who said so.

WALTER M. SMITH,* M. AM. SOC. C. E.—The diversion of water at Chicago has little bearing on the St. Lawrence Waterway.

Three days during the past year (1925), the water flowing through the Chicago River and the Drainage Canal has been turned back into Lake Michigan because of heavy rainfalls. Generally, an increase in the number of typhoid fever cases occurred in the neighborhoods supplied by the water. The first case, which occurred in the spring, was at the mouth of the Calumet River. Before the Health Commissioner could issue orders to boil all the water drawn from the crib located nearest this point, the damage had been done. The boiling had to be continued for about two weeks, just because of only a few hours' change in the direction of the flow of the river. Later, there was a very heavy rainstorm in the center of the city, and the flow of the Chicago River again was reversed, this time for about six hours. Immediately, similar orders went out as regards boiling the contaminated water. As a rule, people observe these directions rigorously because they know the danger. If Chicago should be deprived of the use of this lake water, the city could not exist for very long. It would have to be abandoned, temporarily at least.

* Chf. Designing Engr., Div. of Waterways, Dept. of Purchases and Constr., State of Illinois, Chicago, Ill.

The City of Chicago is now building sewage purification plants as rapidly as they can be constructed. Possibly, engineers are not familiar with the fact that many of these plants are already in existence. It is hoped to build them with sufficient speed so that it will never be necessary to divert more than 10 000 cu. ft. per sec. from the lake. Several years ago when the question of diversion was under discussion, the International Waterways Commission wanted to give the City of Chicago the right to divert 10 000 cu. ft. per sec., and it so recommended. Some of the representatives of the United States and of Chicago objected at that time, because they thought it might be necessary to use more than this in the future, and they did not want such limitation written into the treaty.

The discussion of this paper, with one exception, has been entirely on the question of the profit of building the St. Lawrence Canal from the standpoints of navigation and of water power. Engineers, however, should take a broader view of this subject, and realize that they are dealing with the lives and the property of 3 000 000 people in the question of the diversion of this water from Lake Michigan. It is more than a financial matter. Lake Michigan is an American lake. There is not one drop of water entering it that comes from Canadian soil. All the other lakes are along the boundary line, midway between the two countries.

It seems to the speaker that if the question were submitted to an International Court of Law, the decision would be that the United States Government could divert any quantity of water that it cared to from Lake Michigan, without reference to Canada.

There is still another problem for the City of Chicago in the diversion of lake water. When this question first was agitated, the State of Illinois thought that it was the only party concerned, and that neither Canada nor the United States at large had anything to do with it. For a long time this view seemed to be accepted. The State of Illinois, before it would allow Chicago to divert water and reverse the flow of the Chicago River, passed a law that it must divert 3.33 cu. ft. per sec. per 1 000 of population. Now, if Chicago complies with the State law, it must take about 10 000 cu. ft. per sec. from the lake; if it is limited to the quantity allocated by the United States, it may take only 4 167 cu. ft. per sec. The City must break one of the laws.

RELATION OF THE OHIO RIVER AND ITS TRIBUTARIES TO TRANSPORTATION IN THE UNITED STATES

A SYMPOSIUM

Discussion*

By Messrs. R. N. Begien, L. D. Cornish, Charles S. Churchill, Charles Wuest, Jr., C. E. Grunsky, Charles A. Wilson, and Hunter McDonald.

R. N. Begien,† M. Am. Soc. C. E. (by letter).‡—Prior to the era of railway building, the Ohio River, then a navigable stream during certain parts of the year, was the principal means of transportation and communication for a large tributary territory. As the railway lines were extended across the mountains to Ohio River points, river transportation came to be used conjointly with railway transportation in serving this territory and in reaching communities which were without a railway. As the railway system of the United States developed, competition between the river and the railways grew apace and the joint rail and river transportation was utilized less and less, but it has never been entirely superseded and is used to good advantage in some instances at the present time.

The construction of the Chesapeake and Ohio Railway to Huntington, W. Va., on the Ohio River, was completed January 29, 1873. From that time until August, 1889, when the extension of the road to Cincinnati, Ohio, was placed in operation, the Ohio River formed the connecting link for Chesapeake and Ohio traffic between Huntington and Cincinnati. The river was thus used as an adjunct to the railway in reaching this important city.

At present, coal from inland mines is shipped in some volume by rail to river points and then moved by barge and towage to industries which the originating railroad does not reach, either by its own rails or by satisfactory through rates with other lines. In this manner river transportation is still being used as a supplement to railroad transportation, and to the advantage not only of industries but of individual railroads.

When the Chesapeake and Ohio was extended to Cincinnati, there immediately arose a sharp competition between the railway and the steamboats on the river. This competition still prevails to some extent, for the influence of the river has been, and still is, a factor in the determination of rates on

* This discussion (of the Symposium on the Relation of the Ohio River and Its Tributaries to Transportation in the United States, presented at the Annual Convention of the Society, Cincinnati, Ohio, April 22, 1925, and published in October, 1925, *Proceedings*) is printed in *Proceedings* in order that the views expressed may be brought before all members for further discussion.

† Vice-Pres., in Chg. of Operation, C. & O. Ry., Richmond, Va.

‡ Received by the Secretary, July 8, 1925.

coal. On other business, however, the effect of this competition is negligible and is virtually ignored in present practice.

At one time quite low rates on local merchandise and miscellaneous freights along the Ohio River were made to meet the competition of the irregular steamboats, but it was found that reliable service was of greater importance than the lower rates. Accordingly, rates which were thought to be fair and reasonable were established and since then, notwithstanding the operation of some river steamboats, the local business along the river handled by the railway has continued to increase and the railway has retained its share of the competitive business through superior service.

During the past fifty years a number of steamship companies of apparently ample financial strength and other resources have engaged in transportation on the Ohio River, but they have dropped out one by one and now only a few of them are operating as common carriers. The territory is restricted, the cost of transferring freight from cars to boats and *vice versa* is high, and comparatively few commodities can be handled advantageously. These factors combined with the hazards and seasonal character of river transportation and the competition of efficient railroad transportation at low cost have reduced the river transportation to a minimum.

It is well to emphasize that in the development of American industry reliable transportation has been found of such great value as to constitute an essential. Dependability is the important thing. The transportation afforded by the railways, at somewhat higher cost perhaps, has proved to be of greater economic value than that provided by the less dependable and more restricted inland waterways.

The traffic handled on the Ohio River, excluding that moved from shore to shore by the ferries, for the period 1918 to 1923, inclusive, is as follows:

Year.	Tons.
1918	6 171 413
1919	5 004 337
1920	9 382 463
1921	7 307 880
1922	6 291 825
1923	8 280 520
Total	42 438 478
Average	7 073 080

The tonnage of 1923 was made up as shown in Table 16.

It is apparent that the volume of business is subject to quite wide fluctuations, this being brought about by variations in the total volume of available traffic; by the number and extent of interruptions to navigation by low water, ice, and floods; by railroad competition; and, moreover, by railroad congestion.

Of the total tonnage for 1923, minerals (coal) constituted 90%, ore, metals, etc., 5%, and chemicals 2½%, leaving 2½%, or about 200 000 tons of other freight. Assuming 20 tons per car this would amount to 10 000 car loads. Any one of the great railroad systems which serve the Ohio River territory will load this number of cars in from one to three days.

Under favorable conditions river transportation is well adapted to handling certain tonnage which moves in large volume to consuming industries located on the river banks. The Monongahela River, for example, is used by the steel mills and other industries in the Pittsburgh District for transporting coal from the mines to the plants. This coal is dumped by the mine tipples into barges and is unloaded from them by bucket conveyors or other mechanical devices, so that transportation is afforded at low cost to the users; but even in this case there are substantially no common carriers in operation, and the river bears the same relation to the users as an industrial railroad bears to the industry which it serves.

TABLE 16.

Commodity.	Tons.	Percentage.
Minerals (non-metallic)	7 483 126	90.4
Ores, metals, etc.	889 792	4.7
Chemicals	208 069	2.5
Wood and paper	141 673	1.7
Vegetable food products	35 068	0.4
Animals and products	13 392	0.2
Machinery and vehicles	3 531	0.1
Textiles	2 041	
Miscellaneous	3 828	
Total	8 280 520	100.0

In the Annual Report of the Chief of Engineers, U. S. Army, 1924, the following items in connection with the Ohio River improvements will be found:

Net amount expended on all projects to June 30, 1924	$76 631 552
Total appropriations to date of this report	86 406 577
Amount estimated required to be appropriated for completion of existing projects	24 309 600
Total estimated to complete projects (appropriated and to be appropriated)	110 716 177

Interest on the total estimated cost of the completed projects at 4½%, amounts to $4 982 228 per annum. The cost of operation and maintenance of the Ohio River locks and dams for the fiscal year ending June 30, 1924, was $1 420 575. Assuming a proportionate increase in these costs as the project is continued, the amount per annum on completion will be $2 052 427; adding the interest to this it will be found that a total annual charge of $7 034 655 will be required by the Ohio River waterway.

It has not been practicable to make a reliable estimate of the cost of handling miscellaneous freight by river transportation because of the lack of accurate information and the quite wide variation in the methods followed by the different users of the river. Believing, however, that some comparison of the cost of river and rail transportation would be of interest, an effort has been made to determine the cost of handling coal, which amounts to 90% of the

river traffic, from inland mines to a river point in the vicinity of Huntington, thence by river to another river point in the vicinity of Cincinnati, and also by river and rail to an inland point beyond, as an example of which Indianapolis, Ind., has been selected. Although admitting the probability of considerable variation in the cost of dumping, towing, and elevating coal, it is thought that the estimates given in Table 17 are sufficiently accurate and representative for the purpose.

TABLE 17.

	To river point (Cincinnati) per ton.	To inland point (Indianapolis) per ton.
Railroad rate, mines to river	$0.50	$0.50
Dumping cost (estimated)	0.06	0.06
Towing cost "	0.55	0.55
Elevating cost "	0.25	0.25
Railroad switching rate	0.25	
Railroad rate, river to destination		1.04
Total	$1.61	$2.40
Railroad rate, mines to destination	$1.89	$2.52

This indicates that a probable saving by river transportation of 28 cents per ton to river points, rail delivery, and of 12 cents per ton to inland points can be made. For river-side industries the saving would be about 50 cents per ton. Any large increase in this tonnage would undoubtedly have to go principally to inland destinations. Assuming 20 cents as the average, the river traffic would have to reach a total volume of approximately 35 000 000 tons for the saving to equal the interest charges on the capital expenditure and the annual cost of operating and maintaining the river improvements. This is four times the tonnage of the peak year and about five times that of the average six-year period ending with 1923.

An effort has been made to ascertain what effect the lower cost of river transportation has on the prices of coal in the communities along the river having both rail and water transportation. No reason could be found for believing that the river has any appreciable influence. The price of coal, both retail and wholesale, seems to be fixed on the basis of the rail rates, so that any saving in the cost of transportation by river, as compared with by rail, is not transmitted to the consumer.

The formation of ice in the Ohio River during severe winter weather renders it ineffective as a means of transportation for one to three months at a time. During such interruptions the burden of handling the tonnage normally moved by the river is thrown upon the railroads. It is during severe weather that the railroads are subjected to the greatest load, not only because the effectiveness of locomotives is from 20 to 50% less than during summer weather, due to increased resistance and the greater demands on the boiler, but also because the consumption of coal and other fuel for industrial and domestic uses is greatest during these seasons. If, therefore, the railroads are

to handle the normal river traffic under such conditions, it will mean either one or two things:

1.—That they will have to provide facilities, consisting of locomotives, cars, engine-houses, shops, yards, etc., to take care of this business for a short time each year, or at intervals of two or three years, the cost of which in interest, taxes, and maintenance charges will have to be borne by the other traffic through a higher general average of rates.

2.—If such additional facilities are not provided, the normal railroad traffic increased by the tonnage from the river during severe winter seasons will so tax the railroad facilities that the entire railroad transportation system in the river territory will become congested, traffic will be seriously delayed, the cost of handling it will be unduly increased, and industry and business will suffer. At the same time the capital expenditures involved in the river improvements and the steamboat and other shipping facilities will lie idle. This means, of course, that the rates for river transportation must be high enough to cover the fixed charges and operating costs of the river transportation companies during these periods of enforced idleness.

There is little doubt but that through the proper development of their facilities all the transportation required in the territory served by the Ohio River can be provided by the railroad alone at less cost than the same transportation can be provided jointly by the river and the railroads. It is true that by far the greater part of the capital required for river development is furnished by the United States Government, and that the comparatively few industries and individuals who make use of river transportation contribute a very small part toward the interest and carrying charges of such improvements. Nevertheless, these charges have to be borne by some one, and, in the last analysis, they constitute a burden in the form of taxes on industry in general, the railroad industry not escaping its share.

L. D. CORNISH,* M. AM. SOC. C. E.—There is an important improvement that is under construction by the State of Illinois, namely, that of a 65-mile stretch of the Des Plaines and Illinois Rivers to connect the Great Lakes with the Mississippi River System. That improvement runs from Lockport, forty miles south of Chicago and the terminus of the Chicago Sanitary Canal, down to La Salle, which is the head of navigation on the Illinois River. The Illinois Waterway will have locks of the same dimensions as those on the Ohio River, consequently, the tonnage capacity will be 9 000 in one cargo; the capacity of the entire waterway per year is estimated at 60 000 000 tons.

The paper† by Colonel Kutz is a valuable and timely contribution on the subject of inland waterway transportation. In the past much has been said about the engineering design or construction of waterways, but not enough about the economics of waterway transportation. Such inland waterways as this country possesses have been obtained against the persistent opposition of powerful interests that now maintain that the improvement and maintenance of waterways is unsound, that waterway tonnage is decreasing, and "that all

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† *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1642.

the canals and other inland waterways planned would not make a dent in the American transportation problem", to quote from a recent statement* by Mr. Charles H. Markham.

A summary of Columns (3), (4), and (5) of Table 3† of Colonel Kutz's paper shows that the tonnage in 1924 was 31% greater than in 1913, and 10% greater than the average for the last five years. This indicates a very healthy condition in view of the fact that the main line, the Ohio, is not only incompleted as to its total length, but has several widely separated incompleted sections which prevent the economical development of through transportation. If statistics should be gathered to show the amount of money that has been expended by private capital in the past three years, and is planned to be spent in the next five years, in the development of terminals and floating plant for the Ohio River System, the total would give a better indication of the future of transportation on the Ohio than can be secured from tonnage statistics of the past and present.

In any question of national importance, much depends on the point of view, and this is true of waterways. Many advocates see only the waterway side; its opponents, only the railroad side, or the point of view of materialistic capitalists. The latter ask, Will it pay interest? If not, away with it. This is the view expressed by the National Waterways Commission in the paper from which extracts have been quoted by Colonel Kutz, who expresses a similar thought when he considers the right of the people to "have the products of the farm, mine, and factory move from origin to destination by the cheapest means".‡ Note his use of that little word "move", then reflect on the periods of railroad congestion during the past twenty years when products could not be moved, or could only be moved so slowly that thirty or sixty days' credits were due before the goods were received. Also, reflect on the fact that tonnage doubles every decade, and then try to estimate what the transportation needs of this country will be thirty years hence.

Professor Luther§ forecasts the future national prosperity, and places transportation as the most cardinal requirement. It is the speaker's belief that thirty years hence the need will be so great that no method or artery of transportation will be considered uneconomical. If such be only partly true, then it is the duty of this generation to give proper consideration to the needs of the next, and to continue improving the principal waterways. If such standards had prevailed in the last century as prevail to-day, the United States would not be the leading nation of the world, because one-half of the present railroad mileage would probably never have been built, for fear of the railroads becoming bankrupt. In fact, about two-thirds of those railroads did become bankrupt. Yet who would dare assert now that the construction of those thousands of miles of railroads was not justified?

If such standards should govern waterway improvement, they should also govern highway construction. Fortunately, the people of the State of Illinois

* *Chicago Daily News*, February 23, 1925.

† *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1645.

‡ *Loc. cit.*, p. 1658.

§ *Loc. cit.*, p. 1659.

and its Chief Executive do not judge the needs of that State by those standards. If they did, Illinois would not be, as it is to-day, the leading State on hard road construction. The State has approximately 3 000 miles of hard roads completed and expects to build 5 000 more. Judged by economic considerations of interest, maintenance, and operating cost, it may be unsound, but judged by the need of the people of Illinois, to move, not only their goods, but themselves, they feel that it is justified.

CHARLES S. CHURCHILL,* M. AM. SOC. C. E.—The paper by Colonel Kutz,† brings out many points of interest and value in connection with the great improvement work on the Ohio River; and the tabulated statements, tables, and diagrams contained therein have enabled Messrs. Alfred‡ and Begien§ to make correct and careful comparisons between the true costs of such improved transportation and the corresponding costs of railroads. On this account, as well as many others, the paper by Colonel Kutz is to be commended. The discussions of Messrs. Alfred and Begien contain very reliable statements of railroad statistics which have been compared with the statistics and statements of Colonel Kutz, and deserve the careful consideration, not only of men engaged in, and users of, river transportation, but also an equally careful consideration of men engaged in, and users of, railroad transportation.

When considered in connection with the discussions of Messrs. Begien and Alfred, the paper may become of great use to those officials of the Government who are engaged either in the improvement of waterways, or in the recommendations of further improvements, and especially to citizens who pay their individual share toward the cost of such improvements and who have a right to know that the expenditure is in all particulars a useful one.

That there were no unanswerable objections to these improvements of water transportation on the Ohio River itself seems to be proved by the near and successful completion of the work. The undertaking had a general public approval, and it was not made a political question. However, in the past, some of the improvements by the Government on the smaller rivers have been made on political solicitation and at times against the best judgment of the Government engineers. Some improvements of this kind were made on branches of the Ohio River, and are included in Tables 3‖ and 5¶ presented by Colonel Kutz. The speaker refers to some of this work as being of vital interest to the public, both as to the mistaken local or political reason for its undertaking; the consequent small present public use, and also the great relative fixed charges, plus maintenance and operation costs, and consequent loss of public funds; and the importance of preventing other such wastes of Government funds.

In the spring of 1911, the speaker had occasion to interview J. G. Warren, Lt.-Col., Corps of Engineers, U. S. A. (now Col., U. S. A. (*Retired*)), M. Am.

* Vice-Pres., in Chg. of Purchases, Real Estate, and Valuation, N. & W. Ry., Roanoke, Va.

† *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1642.

‡ *Loc. cit.*, p. 1663.

§ See p. 1883.

‖ *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1645.

¶ *Loc. cit.*, p. 1646.

Soc. C. E., who was then Chairman of a Board of three Army Engineers detailed to pass on the question of the improvement and enlargement of the bridge of the Norfolk and Western Railway crossing the Ohio River at Kenova, W. Va. After this business matter was finished, mention was made of the expenditure of moneys on rivers smaller than the Ohio. Colonel Warren became somewhat reminiscent and on this subject he said in substance, that when he was a Lieutenant and afterward as Captain of Engineers, his recommendations against the advisability of an improvement on such a river amounted to something. Later, with his higher rank, it was often not worth a tinker's dam. In explanation, Colonel Warren stated that politics had often governed the appropriation of Government funds directly against the recommendation of the Government engineering official in charge of the District. One of these cases was the improvement of the Big Sandy River. Another case was a branch of the Ohio River in Kentucky.

Later, the writer met Colonel Warren's Assistant Engineer, at one time connected with the Norfolk and Western Railway, who stated that he had just completed the determination of the maintenance costs of the Erie Canal and had found that these costs were higher per ton-mile than the rates being actually received by the railroads of New York State for carrying the same kinds of freight. These data were regarded by him as proof of the lack of wisdom in recommending the expenditure of large sums of money on canals, especially in territory where railroads exist.

Colonel Kutz presents Table 5 which shows that the fixed charges alone, in mills per ton-mile, on the Ohio River and tributary improvements averaged 4.6 mills. The large sections of the Ohio itself, together with the Monongahela and Allegheny, averaged 4.2 mills; the Big Sandy, however, was 211.0 mills per ton mile; the Little Kanawha River, 29.7 mills; the Cumberland River, 24.5 mills; and the Muskingum River, 16.1 mills. By making use also of the data in Table 3, which gives the cost of "operation and maintenance" in mills per ton-mile, the total of fixed charges, maintenance, and operation, for the Ohio, Allegheny, and Monongahela are found to have averaged 5.88 mills per ton-mile; whereas the Big Sandy was 288.0 mills; the Little Kanawha, 51.3 mills; the Cumberland, 31.1 mills; and the Muskingum, 24.5 mills.

As the economic value of any great engineering work or improvement can be determined only by a complete knowledge of the fixed charges and the maintenance and operation costs of the entire plant, it would have been better if Table 5 had been made complete by the inclusion of all these items.

Charles Wuest, Jr.,* Assoc. M. Am. Soc. C. E.—Having been engaged in railroad as well as river improvement work, the speaker feels that at present his is a neutral position in a discussion concerning the relative merits of railways or waterways as freight carriers. As engineer for a corporation producing about 200 000 tons of paving brick per year, he now has the viewpoint of a contributor to the shipping wants of the carriers.

Under the head of "Economic Comparisons",† Colonel Kutz states:

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* Chf. Engr., The Peebles Paving Brick Co., Portsmouth, Ohio.

† *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1654.

"If the points of origin and destination and the ton-mileage of each commodity hauled on these rivers are known, as well as the water haulage costs, the direct benefits or savings resulting from their use can be readily ascertained by comparison with existing rail rates, and their economic justification determined by comparing the savings or benefits with the fixed charges."

The question that presents itself is, To whom are these direct benefits or savings to accrue? It is here conceded that waterways afford an easy, cheap, and most natural means of moving heavy, bulky freight, and that the development of the rivers should be encouraged to help bring about a general reduction of the prevailing high railroad freight rates. Let us see, however, how far the latter is true at present. On the basis of recent experiences, there seems to be little encouragement in the action of several operating companies on the Ohio River to produce the much hoped-for low freight rates, because of low transportation costs.

The transportation companies, in quoting rates for the transportation of freight by river are not blind to opportunities afforded them by a beneficent Government in opening a way for the conduct of their business, but are totally indifferent about benefits which should rightfully, and, at least, in part, accrue to the public which provides for the way.

In April, 1925, the company which the speaker represents had a prospect of shipping 8 000 tons of paving brick from Portsmouth, Ohio, to Madison, Ind. Having facilities for shipping by river, as well as by rail, the company requested several transportation companies operating on the Ohio River for rates by barge delivery, in expectation that a low and favorable rate by water would insure the movement. The quotations, with certain restrictions of time in loading and unloading attached, were the same in all cases. The rate quoted was $2 per ton.

Consider the fairness of such a rate in view of what has been said about the costs of transporting coal by water. As coal and paving brick differ little in the classification of bulky, heavy freight, there should be little, if any, difference in actual transportation costs.

Colonel Kutz has stated* that, based on the following conditions, 250 miles haul, no back haul, 6 000-ton cargoes in 1 000-ton barges, and wide river and large tonnage, 2.5 mills per ton-mile was a fair measure of haulage cost by waterway. This is practically confirmed by Mr. Alfred,† who has placed the cost at 2.7 mills per ton-mile.

By river, the distance from Portsmouth to Madison, is approximately 200 miles. To be conservative, by using the larger of the two figures, the cost of transporting heavy freight for this distance would be 54 cents per ton. The rate asked was $2. This carries with it 270% for profit and overhead. Why?

On other occasions in the past, rates were requested of the same companies for shipments under similar conditions from Portsmouth to Cincinnati, Ohio. As in the previous instance, a rate of $2 per ton was quoted, in one case, equal to, and, in another, in excess of, competing rail rates between the latter points. This distance by river is approximately 110 miles. From Colonel

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1642.
† *Loc. cit.*, p. 1663.

Kutz's Fig. 4,* the cost per ton-mile for this distance is shown as approximately 3 mills, therefore, the actual cost of transporting 1 ton of heavy freight, based on Colonel Kutz's specifications, between these points would be 33 cents. The rate quoted was $2 per ton, which carries with it 506% for profit and overhead. Why?

This would seem to indicate that the river transportation companies are basing their rates to river towns on rail competition, or as near to it as they deem safe. It would indicate, too, that this is the reversal of a condition mentioned by J. H. Bernhard, Assoc. M. Am. Soc. C. E., in referring to the practice of rebating arising from "the unjust practice of railroads in basing rates to river points on water competition."†

In conclusion, it may be said that a regulation of such conditions is in order, and that fair rates for river hauling must be established if the work of making the rivers into navigable waterways is to proceed unchallenged.

The speaker wishes to express his disapproval of the practices cited, which he believes tends to destroy the real purpose of the splendid work being done by the U. S. Corps of Engineers in the improvement of the National waterways.

C. E. GRUNSKY,‡ PAST-PRESIDENT, AM. SOC. C. E.—This discussion indicates the attention which engineers are to-day giving to the economic aspects of their problems.

What would this section of the Middle West be like if there were no Ohio River and if this river had not fixed the transportation rates? The railroads have been compelled to take the water competition into account. In this connection a proposed canal comes to mind which was under consideration about thirty years ago but was never built. It was an easy matter to show that if this canal had been built, as proposed, for the purpose of moving grain crops out of two counties, it would have compelled a reduction in railroad freight rates which, from the standpoint of the two counties mainly involved, would have justified its construction even if it was never actually used for freight transportation. It might have been unfair to the railroads. Some one would have had to pay the cost.

It is only by a thorough study of the economic aspects of such projects that the question of undertaking them can be satisfactorily answered. The economic problems are equally as important as those of an engineering character.

CHARLES A. WILSON,§ M. AM. SOC. C. E.—These papers and the relative discussion raise the question as to rates for transportation by water and rail and the basis on which such rates are now fixed in the United States. The Interstate Commerce Commission has always been inclined to base rates on the cost of the service and, in this, it has been abetted by Congress. It could depend perhaps on no other basis for support by the judiciary of the Government.

* *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1654.

† *Transactions*, Am. Soc. C. E., Vol. LXXIX (1915), p. 946.

‡ Cons. Engr. (C. E. Grunsky Co.), San Francisco, Calif.

§ Cons. Engr., Cincinnati, Ohio.

The complaint of Mr. Wuest* against the water carrier was based on his desire to have the value of the service considered in fixing the water rate. Great efforts have been made before commissions in rate cases to have the value of the service considered. To-day, there is no rule or basis for fixing rates and, in the speaker's opinion, there is no scientific basis for fixing rates alike for all carriers. If the law of supply and demand is sound economics, then the value of the service should be considered.

The railways running along the valley of the chain of the Great Lakes are apparently successfully competing with the water carriers using the lakes. The tonnage of the carriers using the Great Lakes is tremendous, yet there is no serious competition between the lake carriers and the railways. This water transportation carries very little between producer and consumer except by additional services of the rail carrier.

Rail transportation and water transportation can never be considered as competitive. Highway and waterway transportation differ in their functions and capacities for transportation service and if any are to be governmentally regulated all should be regulated by the same agency and that service assigned which each is fitted by its nature to perform best and most economically.

The comparisons disclosed by the papers and the discussion show no cost for the use of waterways, and without such charges a comparison with the railways is not a fair one. If the railway, through the special characteristics of its type of transportation, can deliver a car at the warehouse of the shipper (and that service is impossible for the waterway carrier), then before a fair comparison can be made between water and rail transportation the cost of delivering the water-borne freight from the waterway to the store door must also be added to the cost of water transportation. One of the principal functions of the Government is to deal justly between all its citizens, but to encourage some by charter to build railways from their private funds and then for the Government to build other ways, such as highways and waterways, out of public funds and give their free use to other citizens, is neither just nor fair and not to be upheld, at least by fair-minded men. The water carrier is forced to use the rail carrier in gathering and distributing the freight constituting the bulk of its tonnage.

The United States Government through its War Department is improving the rivers and seems to be fostering their use in transportation, but in this latter is ignoring the Interstate Commerce Commission. Until all commerce on these rivers comes under the same Government regulating agency, it cannot be expected that the railways will co-operate willingly with either shippers or river carriers in promoting river and rail interchange.

To place waterways, highways, and railways on a parity, the use of the waterway, the highway, and the railway should be paid for by the user, and their use and the charges therefor, placed under the same Government regulating agency. Without such regulation it is neither fair nor right to charge the railway carriers as seeking to discourage the development of waterways because the railways do not co-operate with river carriers.

* See p. 1890.

HUNTER McDONALD,* PAST-PRESIDENT, AM. SOC. C. E.—The Cumberland River Terminal, already mentioned, was completed in 1920 out of funds produced by a bond issue of the City of Nashville of $300 000. Its cost consumed practically all the bond issue. It was built on faith that transportation on the Cumberland River would greatly increase in the near future.

A study of the Cumberland River reveals the fact that above Nashville and for some distance below, the river flows through a deep gorge which it has eroded through limestone rock. It is bounded, therefore, on each side by rocky hills some distance apart. With the exception of the Tennessee Central Railroad for a short distance below Nashville, it is not, and never will be, paralleled by any railroad on account of the prohibitive cost of construction due largely to its crooked course. The country through which it flows would not sustain a railroad built at such cost. The coal which lies on the upper waters of the Cumberland is beyond the reach of present navigation, and it is a question whether if the river is made navigable as far up as this coal region it would move by water or over the Cincinnati-Southern Railroad, which crosses the Cumberland at Point Burnside, the present head of navigation. Such coal as might be moved out by water would necessarily be from mines adjacent to the river.

If it is incumbent on the Government to furnish transportation to people living along the Cumberland River who cannot hope for rail facilities, then the canalizing of that river for transportation purposes only might have been justified; but it is not the duty of the nation to furnish at public expense transportation to people living along rivers any more than it is to furnish transportation to communities living along the non-productive branches of railroads. On many such branches, the cost to the present operating companies is such as to require the expenditure of from $2 to $5 for every dollar that is earned. Nevertheless, solvent roads are required in most cases by public authority to continue the operation of these unproductive lines, the burden falling on the stockholders of the carrier as long as the carrier remains solvent. Furthermore, such solvency often depends on the profits derived from through business.

The traffic on the Cumberland River above Nashville is handled by two small steamboats. From Nashville to the mouth of the river, only one boat operates regularly and urgent appeals for support of this line have been made to the citizens of Nashville, such appeals being based upon sentiment and the desirability of controlling competitive railway rates by water competition.

The upper floor of the terminal warehouse at Nashville was rented shortly after its completion and is still rented to a wholesale grocery firm in an effort to meet a part, at least, of the interest on the bonds. It is entirely proper that Nashville should have provided a terminus to take care of the Upper Cumberland trade, but, in constructing the present facilities, the future has been largely and erroneously anticipated.

Mr. Alfred has referred† to some extent to conditions in Europe with respect to water-borne freight as compared with those in the United States,

* Chf. Engr., N. C. & St. L. Ry., Nashville, Tenn.

† *Proceedings*, Am. Soc. C. E., October, 1925, Papers and Discussions, p. 1663.

but he did not go far enough into the reasons for the existing sentiment in America favorable to freight moving by water. American tourists who travel over Europe come back much impressed by the amount of water-borne commerce on the rivers and inland canals of Europe; but they are ignorant of the conditions which have brought this about.

The industries located along those inland waterways were established and in a flourishing condition many years before railroads were ever thought of. The political influence of these industries and of the people operating craft on the waterways has been so powerful as to throttle the development of the railroads in those countries and they have been prohibited from charging the rates which might have been remunerative but which would take the business away from the water carriers. The result of this policy has been to force the railroads, through foreclosure, into Government ownership, the railroads in Great Britain and a small proportion of those in France being now the only privately operated railroads.

Shortly after the Federal Government took over the American railroads, instructions were issued by Walter B. Hines, Assistant Director General of Railroads, to the Regional Directors of the Western and Southern Districts, R. H. Aishton and C. H. Markham, respectively, to investigate and report on the advisability of the Government undertaking to establish and operate barge lines on the Mississippi and the Warrior Rivers. These Regional Directors appointed a committee of which the late John Howe Peyton, M. Am. Soc. C. E., at that time President of The Nashville, Chattanooga and St. Louis Railway, was Chairman. This Committee held extensive hearings and in May, 1918, submitted its report, which was adverse to the project, such conclusion being based on data fully set out in the report. Having assisted to some extent in the collection of data, the speaker remembers that the five railroads operating between St. Louis, Mo., and Gulf ports had never been congested and were then in position to move speedily and effectively millions of tons more of freight; that they possessed ample ocean terminals; and that congestion on the Eastern seaboard could quickly be relieved if the Government would order the diversion of traffic originating in the West from the congested Eastern ports to Gulf ports and would provide the necessary shipping to take care of such traffic.

It was the conclusion of that Committee that the establishment of the proposed barge lines and their operation by the Government could not be justified under transportation conditions existing at that time.

Now, it is commonly admitted that the Eastern and Western railroad trunk lines were congested during the World War and that they were taken over by the Government largely as a result of this congestion, but, if the speaker remembers the situation correctly, the cause of that congestion was the fact that there was not sufficient shipping at the Atlantic ports to admit of prompt unloading and dispatch of the goods moved to such ports. Everybody knows that damming up the mouth of a river backs up the river. The condition on the railroads was similar. The congestion at the Atlantic terminals prevented further cars being delivered to the seaports and the traffic gradually backed up into the

inland terminals, blocking them. This explains to some extent how the blocking of the terminals was progressive and how, by measuring the rate, it was possible to predict when a complete blockade would take place.

Personally, the speaker has never felt that the taking over of the railroads by the Government was a necessary step. If the Government had given the railroads a very short stay of action and if Great Britain had come to the rescue earlier and furnished enough ships to move the goods promptly from the seaports, the railroads would have worked their way out in an entirely satisfactory manner and without Government control.

MEMOIRS OF DECEASED MEMBERS

NOTE.—Memoirs will be reproduced in the volumes of *Transactions*. Any information which will amplify the records as here printed, or correct any errors, should be forwarded to the Secretary prior to the final publication.

THOMAS NOTTINGHAM JACOB, M. Am. Soc. C. E.*

DIED MAY 25, 1925.

Thomas Nottingham Jacob, the son of W. W. and Helen Williams Jacob, was born at Washington, D. C., on February 6, 1866. His maternal grandfather, Col. John Stutt Williams, of Virginia, served with distinction in the War of 1812.

Mr. Jacob's early engineering experience was gained by service in minor capacities in the Engineering Departments of the Chicago, Milwaukee and St. Paul Railway Company, the Illinois Central Railway Company, and the Union Pacific Railroad Company, from 1886 to 1891. He was then appointed Assistant Engineer on the Kanawha and Michigan Railroad and was engaged on location and construction work for four years, principally in West Virginia.

In April, 1895, he became Assistant Engineer with the United States Engineer Corps, and was assigned to the Rock Island (Illinois) District, where he remained until November, 1897. His work while with the Government was in connection with surveys and plans for the flood control of the Mississippi River.

Mr. Jacobs left the Government service to return to railroad work with the Wabash Railway Company and, later, with the Illinois Central Railway Company, the Chicago, Milwaukee and St. Paul Railway Company, and the St. Louis Valley Railway Company (now the Illinois Division of the Missouri-Pacific Railroad).

In 1903, he formed a partnership with Frank Hutchinson, M. Am. Soc. C. E., for the general practice of Civil Engineering, with offices at East St. Louis, Ill., and, in 1904, he conducted the field investigations relative to flood protection and control at Kansas City, Mo. This partnership continued until the early part of 1909, when Mr. Jacob was appointed Chief Engineer of the East Side Levee and Sanitary District, which was organized for the purpose of protecting about 150 sq. miles of industrial territory in St. Clair and Madison Counties, Illinois. As Chief Engineer of this District, he worked out plans of flood prevention and drainage, in conjunction with the late John A. Ockerson, Past-President Am. Soc. C. E., as Consulting Engineer, involving an expenditure of about $7 000 000. Mr. Jacob served in this capacity until January, 1914, when he again entered general engineering practice, first, in East St. Louis, and, later, in St. Louis, Mo., making a specialty of drainage and levee work, in which he was quite successful. His practice covered important work in Indiana, Illinois, and Missouri, and, at the time of his death, he was Chief Engineer of the Peru, Ind., Flood Control District.

* Memoir prepared by Baxter L. Brown, M. Am. Soc. C. E.

Mr. Jacob was a man of high moral character and commanded the respect of all with whom he came in contact. On November 29, 1893, he was married to Margaret T. Leonard, of Hugheston, W. Va., who survives him.

He was a member of the Engineers' Club of St. Louis, the St. Louis Railway Club, and the St. Louis Institute of Consulting Engineers.

Mr. Jacob was elected a Member of the American Society of Civil Engineers on May 4, 1909.

WALTER GILL KIRKPATRICK, M. Am. Soc. C. E.*

Died May 8, 1925.

Walter Gill Kirkpatrick was born in Canton, Miss., on September 6, 1859. His early education was acquired in the public schools of his native town. He was also a student of the University of Mississippi for one session, 1877-78.

After filling a subordinate position in 1881 with a surveying party of the Denver, South Park, and Pacific Railroad Company, Mr. Kirkpatrick entered Vanderbilt University where he received four degrees, Bachelor of Engineering, in 1886, Civil Engineer, in 1887, Bachelor of Science and Master of Science, in 1889, and was for two years Fellow and Assistant in Engineering.

He spent several years in municipal, bridge, and railroad engineering in Tennessee, partly as Assistant Engineer and partly in responsible charge of work, and was then, successively, United States Assistant Engineer in charge of a party surveying the Cumberland River for a system of locks and dams, locating Lock A; Secretary of the Engineering Association of the South; Acting Professor of Civil Engineering, for a short time, in Union College, Schenectady, N. Y.; and Engineer in charge of the Tennessee Centennial Exposition, for a period of six months.

For several years, Mr. Kirkpatrick's work was largely in Mississippi, where he was engaged as a Designing and Constructing Engineer for electric light plants, water-works, and sewerage systems in Canton, Holly Springs, West Point, Crystal Springs, and other of the more important towns of the State. In 1898, he was made the first Municipal Engineer of the City of Jackson, Miss.

In 1899, Mr. Kirkpatrick constructed a Melan arch bridge in Jackson, the first of its kind to be built south of the Mason and Dixon Line and the sixth of its kind in the world. It was only 6 in. thick at the crown, and was built of reinforced concrete. The old surface wore away and a new one has replaced it, yet the body of the structure, enwrapped in the strength given it by its designer, stands to-day and is giving continuous service.

A few years later, he constructed the first concrete sidewalks to be laid in Mississippi. With opposition on all sides, he built these walks confidently and unhesitatingly. He also constructed the first radial brick chimney in Mississippi. It was built in ten days by four men, and is still giving continuous service. He also drew up the first plumbing sanitary ordinances in Mississippi

* Memoir prepared by Alfred Hume, Chancellor, Univ. of Mississippi, University, Miss.

—ordinances which have been adopted by nearly every town of any size in the State and by many towns in other Southern States.

The facts thus briefly enumerated, and others which might be mentioned, abundantly justify the claim that Mr. Kirkpatrick did genuinely pioneer work in his profession and that it was work of the highest grade.

From Jackson, he went to Birmingham, Ala., as Municipal Engineer of that city, after which he was with The Alabama Power Company and then became City Engineer of Monroe, La. Later, still, he was engaged in private practice as a Consulting Engineer in Atlanta, Ga. He served as Designing and Consulting Engineer for more than a score of towns and cities in nearly all the Southern States, building water-works, drainage systems, electric light plants, etc. He also served as Consulting Engineer for a number of the largest Southern cities. For two years prior to his death, he had been Professor of Municipal Engineering at the University of Mississippi.

The following is quoted from resolutions adopted after Professor Kirkpatrick's death by the Faculty of the University of Mississippi:

"As a student at Vanderbilt University, he was conspicuous for his fine scholarship and fine manhood. A student of deep and earnest purpose, he was an inspiration to the younger men. He made it a principle to master each subject from day to day; he hesitated at no task; he found no problem too difficult. His splendid success in no way spoiled the lovableness of his young manhood. The faculty early showed its high esteem by appointing him to a responsible teaching fellowship and by the bestowal of various other honors.

"His later life was of a piece with his college years. Always and everywhere he was a Christian gentleman.

"As a practicing engineer he had the highest regard for professional thoroughness and professional ethics. He worked with a patience and accuracy worthy of all emulation. In Nashville, Tenn.; in Jackson, Miss.; in Monroe, La.; in Birmingham, Ala.; and in many other Southern cities, he held positions of the highest responsibility and held them with honor to his profession. He loved his work.

"As a University professor he was a deep student, an enthusiastic teacher, and a warm friend. Even as a practical engineer he was a valued teacher, for he was always an inspiration to the young engineers under him.

"We mourn his loss,
A beautiful life has ceased to be,
A gentle star has been blotted from the galaxy."

The writer was intimately acquainted with Professor Kirkpatrick from the time of their college days at Vanderbilt University. In a recent report to the Board of Trustees of the University of Mississippi, referring to the Faculty, he said:

"The University has suffered a heavy loss in the death of Professor Walter Gill Kirkpatrick of the Chair of Municipal Engineering. He died on the 8th of May, when the Government Steamboat, *Norman*, sank in the Mississippi River, about sixteen miles south of Memphis. The Engineers of the Mid-South had been holding a two days' session in the City of Memphis, and had been down the river on the fatal day, inspecting Government work. Members of the American Society of Civil Engineers were in conference on the boat at the time of the disaster.

"Although Professor Kirkpatrick had been connected with the University only two years, he had made a profound impression on his students. While

his loss seems simply irreparable, yet the moral and spiritual effect of his tragic death is one of the finest things I have ever witnessed. The affection and esteem in which he was held has created a spirit and atmosphere of incalculable value. I have never known a more admirable man, one whose life was whiter or more straight. He was the very soul of honor. Not a single blemish marked his record. I think that no mere man ever had a more nearly flawless character or led a more blameless life."

The same disaster that caused the death of Professor Kirkpatrick resulted in the the drowning of his wife, *née* Miss Willie Jones, of Canton, Miss. They had no children.

Professor Kirkpatrick was elected an Associate Member of the American Society of Civil Engineers on April 6, 1892, and a Member on October 5, 1898.

CHARLES HENRY MILLER, M. Am. Soc. C. E.*

DIED MAY 8, 1925.

Charles Henry Miller was born at Strasburg, Lancaster County, Pa., on November 30, 1866, the son of Henry B. and Elizabeth (Bartholomew) Miller, who were also natives of Lancaster County. He received his education in the public schools of Strasburg, and was graduated from the High School in 1884 and from Lehigh University, with the degree of C. E., in 1888. Immediately following his graduation, Mr. Miller became identified with the improvement of the Mississippi River under the direction of the United States Corps of Engineers. While in this employ he conducted surveys, dredge work, bank revetment, etc., and held the titles of Instrumentman, Draftsman, Assistant Engineer, Chief of Survey Party, and, finally, Superintendent of Construction, his employment having extended over a period of thirteen years.

In 1901, Mr. Miller was made Superintendent of Construction of the McClintic-Marshall Construction Company at Pittsburgh, Pa., and had charge of the building of the Pittsburgh plant of that Company. He maintained this connection for four years. In 1905 he was made Engineer of River Protection for the Missouri, Pacific, and Iron Mountain Railway System, but resigned this position in 1911, to organize and become President of the Miller Engineering Company, Little Rock, Ark. This Company was engaged in both engineering and construction work, and Mr. Miller had personal charge of the Engineering Branch. During the six-year period ending in 1917, he served as Chief Engineer of the Cairo Levee District, the St. Johns Levee and Drainage District, and the Indian Bayou Drainage Districts Nos. 1 and 2; he was a member of the Board of Consulting Engineers for several railroad companies on river bank protection and drainage work; and Consulting Engineer for the United States Government on the "sunk land" cases in Eastern Arkansas. His Company acted as Contractors for municipal improvements in Batesville and Little Rock, Ark.; built five miles of river-bank protection work for the Caddo Levee Board on Red River, Louisiana; Dam No. 2 in the Ouachita

* Memoir prepared by Alfred M. Lund, M. Am. Soc. C. E.

River, Louisiana; and extensive dike and bank protection work on the Mississippi River, the last two having been constructed for the U. S. Corps of Engineers.

In the midst of these activities as an Engineer and Contractor, Mr. Miller put aside all thought of personal interest on the entrance of the United States into the World War, and enlisted in June, 1917, receiving a commission as Major of Engineers. He acted as Constructing Quartermaster in charge of the building of Camp Cody at Deming, N. Mex. In December, 1917, he was appointed Commanding Officer of the 2d Battalion of the 23d Engineers (Highway Regiment). Major Miller arrived in France in April, 1918, and was placed in charge of railway, highway, camp, motor-park, and hospital construction, dealing with as many as 15 000 troops, including service battalions, prisoners, and Chinese laborers.

In February and March, 1919, he commanded the 2d Battalion of the 307th Engineers of the 82d Division. In April, 1919, he returned to the United States, and received an honorable discharge on April 5, 1919.

On his return to civil life, Major Miller re-organized the Miller Engineering Company as the Miller-Butterworth Company, Engineers and Contractors. This firm immediately launched into the active road-building program of Arkansas and built thirty-three miles of "Warrenite" pavements in Arkansas County. Major Miller still continued to devote his attention to the engineering part of the Company's work. He was appointed by the United States Supreme Court as a Commissioner to decide on twelve miles of the boundary between Arkansas and Mississippi, and he also served on a commission to establish a portion of the boundary between Oklahoma and Texas. He acted as Consultant or as Arbiter in many suits on highway work about which he was recognized as an authority. At the time of his death he was serving as representative of the State of Arkansas on a Board appointed to equalize the taxes of Conway County.

Besides being President of the Miller-Butterworth Company, Major Miller was Vice-President of the Southern Sand Company, President and General Manager of the Allen Gravel Company, and Vice-President of the Southern Material and Construction Company.

Major Miller was also active in scientific and civic affairs. He was a member of the American Association of Engineers, the American Railway Engineering Association, and the Little Rock Engineers' Club, and a Director of the Federated Engineering Council, and the Little Rock Chamber of Commerce.

In 1900, he was married to Edna Ward, of Luna Landing, Ark., by whom he is survived. He is also survived by two daughters, Mrs. Edgar A. O'Hair, wife of Capt. E. A. O'Hair, F. A., U. S. Army, Fort Sill, Okla., and Mrs. Stacy C. Howell, wife of Dr. S. C. Howell, Little Rock, Ark.

Major Miller was very popular with all his associates and acquaintances and highly respected by his fellow engineers. He was generous with his means and his time and always active in any undertaking that concerned the welfare of individuals or of the community in which he lived.

He lost his life on May 8, 1925, when the U. S. Steamer *Norman* sank in the Mississippi River, carrying to their death twenty-three of the passengers and crew. The passengers were engineers and their families in attendance at the Mid-South Convention of Engineers, at that time meeting at Memphis, Tenn.

Major Miller was elected a Member of the American Society of Civil Engineers on May 2, 1899.

GEORGE THOMAS ROBERTS, M. Am. Soc. C. E.*

Died September 2, 1924.

George Thomas Roberts, the son of George and Jennie (Whiteman) Roberts, was born in Buffalo, N. Y., on September 23, 1871. He attended the public schools and the Central High School of that city until 1891.

At the age of 20 years, Mr. Roberts began his engineering work as a Draftsman with the Lehigh Valley Railroad Company at Buffalo. In 1894, he became a Draftsman in the office of the City Engineer, and, later, was appointed Assistant City Engineer.

In 1901, he formed a partnership with F. V. E. Bardol, M. Am. Soc. C. E., who was at that time City Engineer of Buffalo. The organization was called the Eastern Concrete Steel Company, Building and Engineering Contractors. Mr. Roberts continued in that business until his death.

Among the buildings erected by this Company are the New York State Normal School, City Hospital Buildings, Majestic Theatre, Elmwood Theatre, St. Margaret's Church, Canisius College, all at Buffalo; also, several of the Eastman Kodak Company Buildings, at Rochester, N. Y., the Drill Hall at Cornell University, Ithaca, N. Y., and the Public Library at Westfield, N. Y.

He was a member of the Buffalo Consistory, Ismalia Temple, of Buffalo; Washington Lodge No. 240, F. and A. M.; the Buffalo Club, the Athletic Club, the Park Club, and the Automobile Club; and, also, of the Buffalo Builders' Exchange.

Mr. Roberts was elected an Associate Member of the American Society of Civil Engineers on September 3, 1902, and a Member on March 3, 1908.

JOHN STERLING DEANS, Assoc. M. Am. Soc. C. E.*

Died August 18, 1924.

John Sterling Deans, the son of the late John Sterling Deans, M. Am. Soc. C. E., and Clara Barr Deans, was born in Phœnixville, Pa., on July 5, 1891. His father was for many years prominently connected with the Phoenix Bridge Company of Phœnixville, as Vice-President and Consulting Engineer.

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* Memoir prepared by Emile Low, M. Am. Soc. C. E.

Mr. Deans received his Ph. B. from the Sheffield Scientific School in 1912, and was in the employ of the Pennsylvania Railroad Company from 1912 to 1916, first, in the Construction Department and, later, in the Maintenance-of-Way Department.

From 1916 to 1917, he was connected with the National Aniline and Chemical Company as Superintendent of Engineering Construction of the Abbott Road Plant at Buffalo, N. Y.

Mr. Deans enlisted in the Ordnance Corps of the United States Army, and was commissioned a First Lieutenant on October 5, 1917, and assigned to the Construction Section of the Supply Division at Washington, D. C., where he was in charge of the design of railroad and civil engineering work. He was promoted to the rank of Captain in June, 1918, and in the following September was sent to France. There he was assigned to the Construction and Maintenance Division of the Ordnance Department.

Mr. Deans was honorably discharged from the Army in January, 1919, and then entered the construction business as General Superintendent for the J. N. Byers and Son, Incorporated, of Buffalo. At the time of his death, he was Secretary of the firm.

He was married in 1916 to Harriet Gertrude Byers, a sister of Mr. J. Newton Byers. Mrs. Deans survives him, with a son and two daughters; he also leaves his mother, brother, and two sisters.

Mr. Deans was elected an Associate Member of the American Society of Civil Engineers on September 9, 1919.

FRANKLIN JAMES VAN HOOK, Assoc. M. Am. Soc. C. E.*

Died July 26, 1925.

Franklin James Van Hook, the son of Frank and Matilda (Wallace) Van Hook, was born at Lockport, N. Y., on July 31, 1881. Prepared for college in local schools, he entered the Massachusetts Institute of Technology, and was graduated in 1906 with the degree of S. B. (in Civil Engineering).

Employment by the "Big Four" Railroad Company, at Wabash, Ind., the Charles River Basin Commission, the New York State Water Supply Commission on the Racquette River, and the Commissioners of Sewerage at Louisville, Ky., occupied the first five years of Mr. Van Hook's technical career. Later, he was engaged on grade-crossing elimination at Nashville, Tenn., for the Louisville and Nashville Railroad Company.

In 1912, Mr. Van Hook entered the service of the City of Cincinnati, Ohio, in connection with the preparation of a plan of comprehensive sewerage. In this work he made a particular study of rainfall and run-off, and outlined requirements for relief drainage. Following this engagement, he served as Resident Engineer for the Louisville and Nashville Railroad Company on the construction of the First Avenue Viaduct at Birmingham, Ala. During 1915, 1916, and 1917, he was Superintendent of Construction for the Jefferson Con-

* Memoir prepared by Frank C. Tolles, M. Am. Soc. C. E.

struction Company of New Orleans, La. In this capacity, and in addition to other work, he supervised the installation of the Warrior River steam power plant of the Alabama Power Company.

The call of the World War found Mr. Van Hook debarred from active service. His desire to help took him into cantonment construction at Camp Meigs and elsewhere for the Construction Division of the United States War Department. In 1918 and 1919 he was Field Engineer for Westinghouse, Church, Kerr and Company on the construction of the United States Government Nitrate Plant at Muscle Shoals, Ala.

He spent the period from 1919 to 1924 in Cleveland, Ohio, where he served chiefly as Designer and Estimator of industrial buildings for the Boldt Construction Company and the Sam W. Emerson Company.

In 1924, Mr. Van Hook was made General Manager of the Oil City Wood Working Manufacturing Company, Oil City, Pa., and, later, became a Director of the Company. At the time of his death, he was engaged on plans for enlarging the activities of that firm.

Mr. Van Hook was a member of the Baptist Church. He was a Mason, a Rotarian, a Director of the Western Pennsylvania Builders' Supply Association, and held membership in other organizations, both civic and social.

He was married in 1916 to Madge Nave, of Louisville, Ky. Mrs. Van Hook and two children, Robert Wallace and Annie Laurie, survive him.

His career is typical of many who make up the profession and help to formulate and sustain its best traditions. It is a career which subordinates self in loyal devotion to the interests of employer or client. Mr. Van Hook was such an engineer. His abilities were not inconsiderable. His life was useful.

Mr. Van Hook was elected an Associate Member of the American Society of Civil Engineers on May 7, 1913.